Module 4: Addition and Subtraction

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Objective

- To gain an understanding of flexible methods of addition and subtraction and the appropriate time to implement them
- To promote thinking about flexible methods and emphasize the need for a strong understanding of the operations coupled with place value

Tennessee Standards

1.NBT.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

1.OA.4 Understand subtraction as an unknown-addend problem. *For example, subtract 10-8 by finding the number that makes 10 when added to 8.*

1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

2.NBT.7 Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

2.NBT.8 Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.

2.NBT.9 Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawings or objects.)

2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

TEAM Alignment

- Standards and Objectives
- Presenting Instructional Content
- Activities and Materials
- Questioning
- Teacher Content Knowledge
- Thinking
- Problem Solving

Participant Activities:

- Participants will learn student activities to connect place value understandings to addition and subtraction
- Participants will read, reflect, and discuss three types of computation strategies
- Participants will consider ways to support invented computational strategies by students
- Participants will engage in activities that promote reasoning strategies for addition and subtraction of multi-digit numbers

Opening Activity

Let's Warm Up Our Minds!

https://www.youtube.com/watch?v=bsVBonvZuUg&app=desktop

Connecting Place Value to Addition and Subtraction

There is much more to learning about place value than having children state how many ones, tens, or hundreds are in a number. Children need ample time and opportunities to fully understand place value because it is such a complex concept - so complex that it took humans centuries to develop.

Researchers also suggest that problems involving addition and subtraction are a beneficial context for learning place value.

> "We know that children who only understand computation as a digit oriented exercise and not with full understanding of the numbers involved make many errors and have little judgment of the reasonableness of their answers. We must focus on helping children build flexible computational strategies using place-value knowledge. Here we lay the groundwork for developing both conceptual and procedural knowledge as we connect place value to addition and subtraction."
>
> Wright et al., 2008

Reflection

- Consider the scope and sequence you use. Does it align with the message of these quotes?

Student Activity: 50 AND SOME MORE (Supports 2.OA.1 and 2.NBT.7)

Retrieved from www.engageny.org

1. Say or write a number between 50 and 100.
2. Children respond with "50 and _____."
3. Any benchmark number can be used instead of 50. For example, you could use any number that ends in 50. Or as a challenge, use numbers such as 70 or 230.

Student Activity: THE OTHER PART OF 100 (Supports 2.OA.1 and 2.NBT.7)

Retrieved from www.engageny.org

Benchmark numbers are often used in computational strategies to make the computation easier to do. This activity is aimed at what may be the most important benchmark of all: 100.

1. Two children work together with a set of little ten-frame cards.
2. One child makes a two-digit number.
3. Then, both children work mentally to determine what goes with the ten-frame amount to make 100.
4. They write their solutions on paper and then check by making the other part with the cards to see if the total is 100.
5. Children take turns making the original number.

If children are adept at finding parts of 100, you can change the whole from 100 to another number. To start, try other multiples of 10, such as 70 or 80. Then, extend the whole to any number less than 100.

Participant Activity

Suppose that the whole is 83. Sketch four ten frame cards showing 36. Looking at your "cards", what goes with 36 to make 83?

Reflection

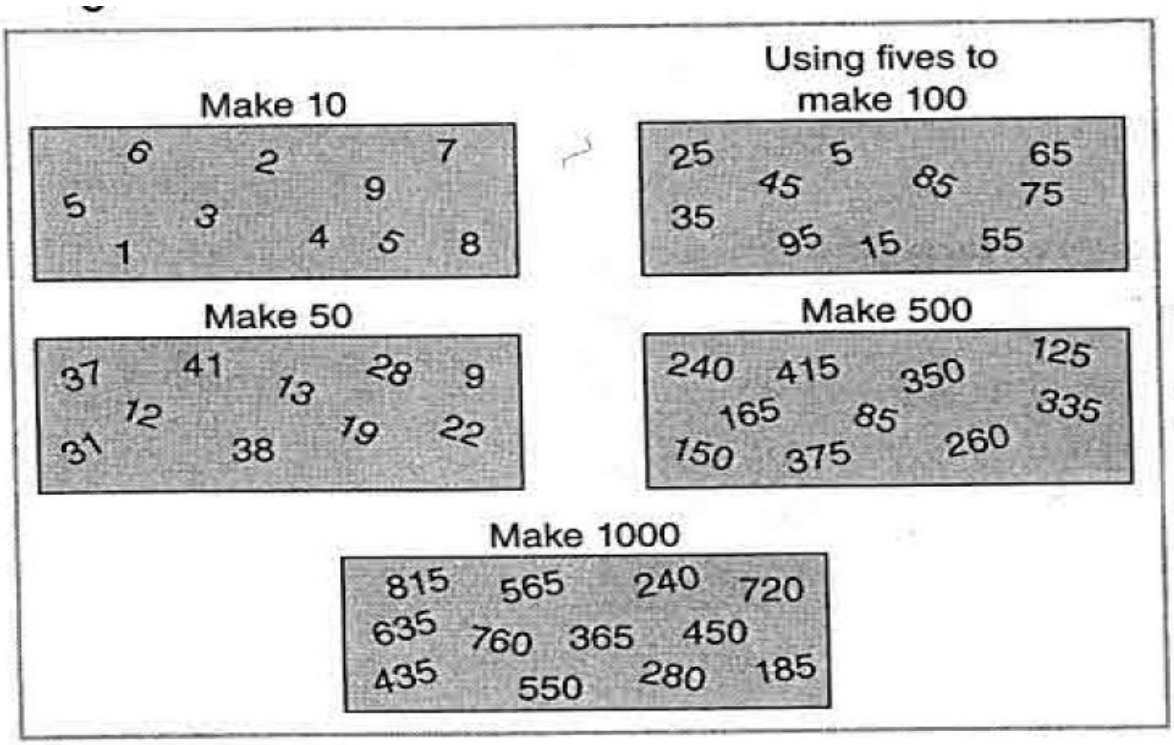
- What process did you go through to find your answer?

Student Activity: COMPATIBALE PAIRS (Supports 1.OA.4, 1.OA.6, 2.OA.1 and 2.NBT.7)

Van de Walle, Lovin, Karp, & Bay-Williams., 2014

Compatible numbers for addition and subtraction are numbers that go together easily to make benchmark numbers. Numbers that make tens or hundreds are the most common examples.

1. Searching for compatible pairs can be done as an activity with the full class.
2. The possible searches are at different difficulty levels.
3. Children name or connect the compatible numbers as they see them.



KEY IDEA #8

Flexible methods of addition and subtraction involve taking apart (decomposing) and combining (composing) numbers in a variety of ways.

Most methods for decomposing numbers are based on place value or compatible numbers.

Student Activity: CLOSE, FAR, AND IN BETWEEN

Van de Walle, Lovin, Karp, & Bay-Williams., 2014

This activity has children apply some of the same ideas about benchmark numbers that we have been exploring.

1. Put any three numbers on the board. Use two-digit numbers if those are more appropriate.
2. With the three numbers as reference, ask questions like the ones listed below. Encourage discussion of all responses.
    - Which two numbers are the closest? How do you know?
    - Which number is closer to ____?
    - Name a number between _____ and _____?
    - If these are "big numbers" what are some small numbers?
    - What are some numbers that are about the same?

Participant Activity

Working in pairs, your facilitator will give you three three-digit numbers. In partners, you will come up with a list of questions that could be used in this activity. Share and chart questions as a whole group.

Close, Far, and in Between
Numbers :
Questions

Student Activity 11 More (1.NBT.5)

11 More

Number of players: 2 **Materials**: Counters of 2 different colors (one color per player), die

Instructions: Players take turns to roll a die and calculate the total of the number rolled plus 11.
After each roll the player places a counter on a corresponding number on the board. For example, if Player 1 rolls a 4 s/he may place a counter on any 15 on the board because 4+11=4+10+1=15. Play continues until one player has 4 counters in a row (horizontally, vertically, or diagonally).

12	14	13	15	16	17
14	12	16	15	13	12
17	12	14	13	15	16
16	15	13	14	12	17
17	16	12	15	14	17

I know that 11 more than ___ is ___ because ___ plus 10 is ___ and 1 more is ___.

10 more than ___ is ___. 11 more than ___ is ___.

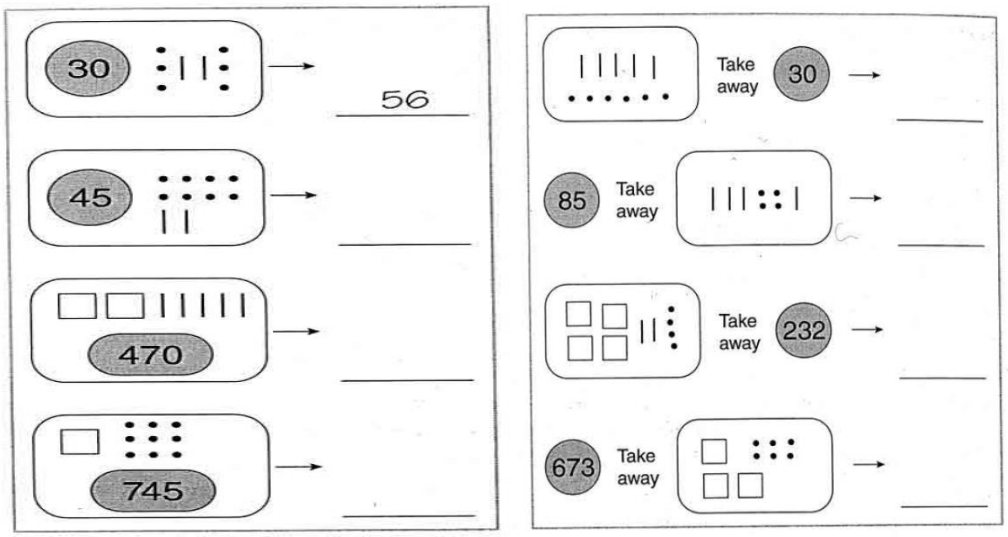
Retrieved from http://www.k-5mathteachingresources.com/support-files/11-more.pdf

Student Activity: NUMBER SQUARES, STICKS AND DOTS (2.NBT.7)

Battista, M. *Cognition- Based Assessment & Teaching of Addition and Subtraction: Building on Students' Reasoning*, 2012

This activity combines base-ten representations with symbolism.

1. On a circle of laminated construction paper, write a number with a dry erase marker.
2. Have students close their eyes as the teacher adds base ten materials. Add them in different orders and arrangements to promote flexible thinking.
3. Once students open their eyes, have them mentally compute the number.
4. Ask students to share their thought process.
5. If done in a small group, students may need to physically touch the pieces.
6. The same activity can be done for subtraction using a "take away" sign
7. You can do this as a whole class or a worksheet. If done as a worksheet, have students write about how they solved the problem. It is still important, however, to have a discussion with the class.



Student Activity: Base Ten Bag: Addition (2.NBT.7)

Base Ten Bag: Addition

Materials: Base 10 blocks (12 tens and 12 ones), paper bag, die

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1. Work with a partner. Partner A: Roll a die and collect that number of tens. Roll again and collect that number of ones blocks.
2. Both partners: Record the total value of the blocks before placing them into the paper bag.
3. Repeat steps 1 and 2 with partner B rolling the die.
4. Figure out the total value of the blocks in the bag. Show your work.
5. Remove the blocks from the bag. Check your work by grouping and counting the tens and ones blocks.
6. Repeat steps 1-5 at least five times.

I think the total value of the blocks in the bag is ___ because ...

Retrieved from http://www.k-5mathteachingresources.com/support-files/base-10-bag-addition.pdf

Student Activity: HUNDREDS CHART ADDITION (1.NBT.4 and 2.NBT.2)

- Conklin, M & Sheffield S., 2010

1. Display a hundreds chart (or thousands chart) for all to see, or give each child their own individual chart.
2. Children use the hundreds chart to add two numbers.
3. There are many ways that children can use the hundreds chart to add two numbers, so class discussions are a must.
4. Have children work on one sum at a time and then have a discussion to compare the different methods.

The hundreds chart can be thought of as a stacked number line - one that emphasizes the distance from any number to the next multiple of 10. To begin, pose problems with relatively small second numbers such as:

17 + 14 23 +12 35 + 13 78 + 15

Many children will initially count on by ones from the first number, which is an indication that they may not understand how to count by tens from a given starting value (an important place-value concept).

Reflection

- How do the previous activities help promote mathematical understanding?

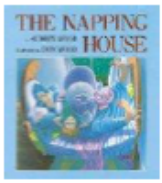
Moving To Computational Fluency

Addition and subtraction strategies that build on decomposing and composing numbers in flexible ways contribute to children's overall number sense. In most everyday instances, these alternative strategies for computing are easier and faster than the standard algorithms and can often be done mentally.

Literature Connection: **The Napping House** by Audrey & Don Wood.

A cumulative rhyme leads up to the consequences of piling too many sleepy people and animals in a cozy bed. The following task may be used after the story has been read.

The Napping House



Materials: copy of *The Napping House* by Audrey Wood

1. After listening to the story solve the following problem:

   When all the sleepers were piled up, how many feet were in the bed?

2. Use pictures, numbers and/or words to show how you solved the problem.

Retrieved from www.k5mathteachingresources.com

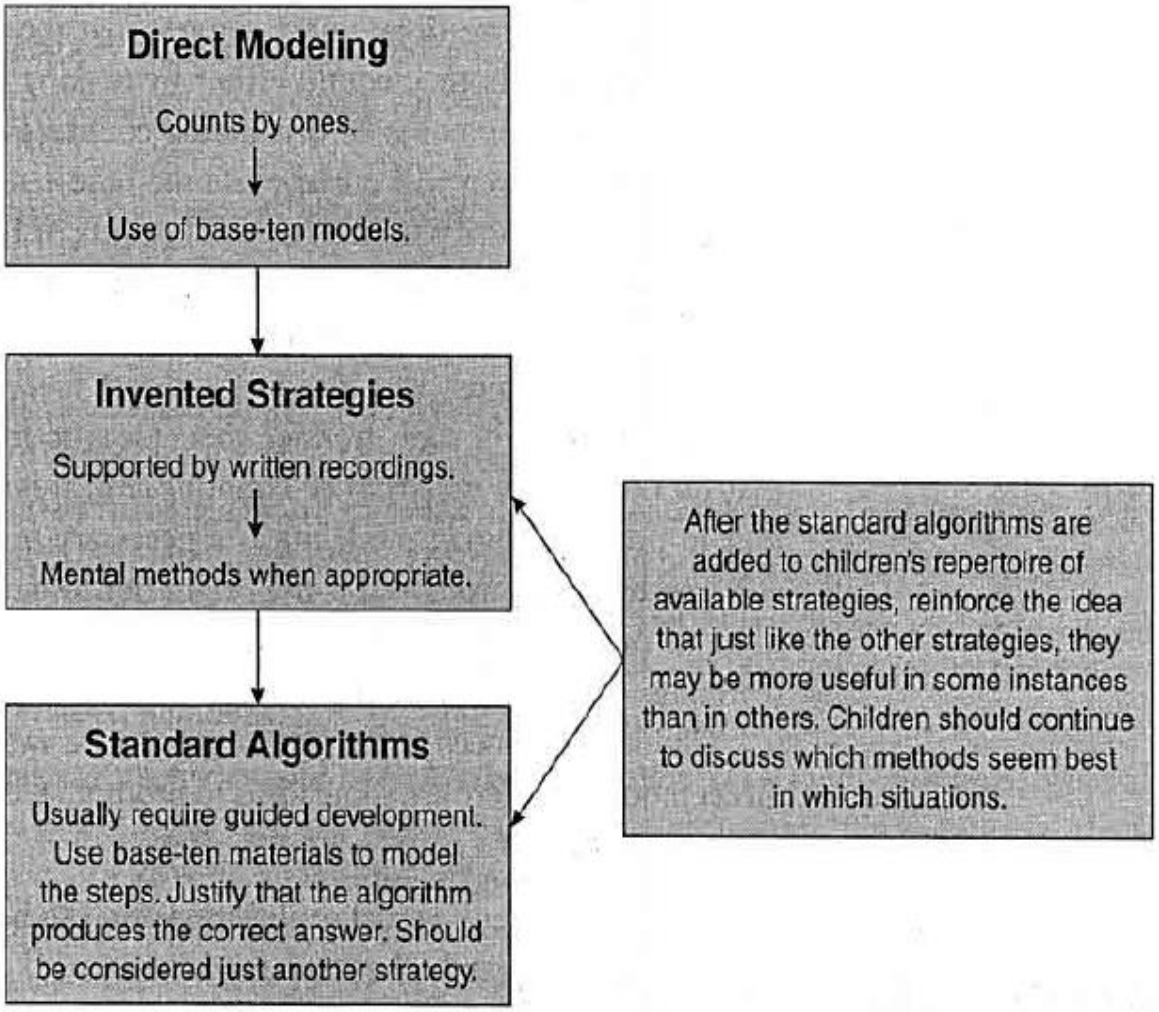
Participant Activity

Complete the following problem. Be prepared to discuss your reasoning with your small group.

Mary has 114 spaces in her photo album. So far she has 89 photos in the album. How many more photos can she put in before the album is full?

Three types of Computation Strategies

Van De Walle notes that there is a general instructional sequence for three types of computation strategies.



Van de Walle, Lovin, Karp, & Bay-Williams., 2014

Direct Modeling

The developmental step that usually precedes invented strategies is called direct modeling: the use of manipulatives, drawings, or fingers along with counting to directly represent the meaning of an operation or story problem. This may be a child who has modeled numbers in a problem using counters then counted by ones to find an answer. Children who consistently count by ones most likely have not developed base-ten grouping concepts.

Children may need encouragement to move away from direct modeling. Here are some ideas to promote the transition from direct modeling:

- Record children's verbal explanation on the board in ways that they and others can model.
- Ask children who have just solved a problem with models to see if they can do it mentally.
- Ask children to make a written numeric record of what they did when they solved the problem with models. Explain that they are going to try to use the same written method on a new problem.

Note: Accepting direct modeling as a necessary developmental phase allows children who are not ready for more efficient methods a way to explore the same problems as classmates who have progressed beyond this stage.

Invented Strategies

We refer to any strategy, other than the standard algorithm, that does not involve the use of physical materials or counting by ones as an invented strategy. Tennessee State Standards describes these as "strategies based on place value, properties of operations, and/or the relationship between addition and subtraction". More specifically, children are expected to "develop, discuss, and use efficient, accurate and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations."

"There is mounting evidence that children both in and out of school can construct methods for adding and subtracting multi-digit numbers without explicit instruction. But not all children invent their own strategies. So strategies invented by class members are shared, explored, and tried out by others. However, children should not be permitted to use any strategy without understanding it."

Parrish, 2010

Contrasts between Invented Strategies and Standard Algorithms

There are significant differences between student-invented strategies and standard algorithms.

1. Invented strategies are number oriented rather than digit oriented. Using the standard algorithm for 45 + 32, children never think of 40 and 30 but rather 4 + 3.

2. Invented strategies represent a range of flexible options rather than "one right way". Invented strategies tend to change with the numbers involved in order to make the computation easier. The standard algorithm suggests using the same approach each time.

Participant Activity

Try each of these mentally. Be prepared to discuss with your group.

465 + 230 526 +98

Reflection

- Did you use the same method to solve each?

Participant Activity

The development and use of invented strategies support procedural proficiency. The positive benefits are difficult to ignore. With your table, generate a list of the benefits of using invented strategies. These will be charted as a whole group.

Invented Strategies

1. **Children make fewer errors**. Students tend to make fewer errors because they understand their own methods. Even when well-explained or modeled, many children do not understand the underlying concepts of standard algorithms. Not only do these children make more errors, but the errors are often systematic and difficult to remediate. Errors with invented strategies are less frequent and less systematic.
2. **Less reteaching is required**. Often teachers are concerned when children's early efforts with invented strategies are slow and time consuming. But the extended time for these early stages results in a meaningful and well-integrated network of ideas that is robust and long lasting. The increase in development time results in a significant decrease in reteaching and remediation.
3. **Children develop number sense**. Children's development and use of number–oriented, flexible algorithms helps them cultivate a rich understanding of the number system, especially place value concepts.
4. **Invented strategies are the basis for mental computation and estimation**. When invented strategies are the norm for computation, there is no need to talk about mental computation as if it were a separate skill. As children become more and more proficient with these flexible methods, they find they are able to use them mentally without having to write down even intermediate steps.
5. **Flexible methods are often faster than the standard algorithms.** Those who become adept with invented strategies will often perform addition and subtraction computations more quickly than those using a standard algorithm.
6. **Strategy inventions is itself an important process of "doing mathematics"**. Children who invent a computational strategy or who adopt a meaningful strategy developed by a classmate are involved in the process of sense making. This development of procedures is a process that is often hidden from children. By engaging in this aspect of mathematics, a significantly different and valuable view of "doing mathematics" is revealed to learners.
7. **Children who use invented strategies perform similarly or outperform their counterparts who are taught only standard algorithms.**

Reflection

- Do you feel the use of invented strategies is effective or ineffective? Why?

- How have you seen this reflected in your classroom and own teaching practices?

Standard Algorithm

Children who already know the standard algorithm may resist the development of more flexible strategies. What do you do then?

First and foremost, apply the same rule to standard algorithms as to all strategies. *If you use it, you must understand why it works and be able to explain it.* In an atmosphere that says, "Let's figure out why this works," children can profit from making sense of standard algorithms just as they can with invented strategies. But the responsibility for the explanations should be theirs, not yours!

As second grade teachers may feel the need to introduce the standard algorithm, it should be noted that students should spend a significant amount of time with invented strategies - months, not weeks. Teachers should begin with groupable base ten models, then go to the pre-grouped model, and last to the non-proportional model. Place value disks can be an excellent tool.

Video : Adding with Place Value Disks
https://www.youtube.com/watch?v=2aE6SmzX_7o

The following video shows how students begin making the connections of what they are doing while adding with the discs to a more abstract pictoral model. This is done after significant time has been spent with the place value disks.

Video: Pictoral Representations
https://www.youtube.com/watch?v=3_r2aFlNzLc

Reflection

How can using this manipulative help a students' mathematical understanding of adding when composing a ten is required?

Mental Computation

What may be a mental strategy for one child may require written steps for another child. Initially, children may not be ready to do computations mentally, as they may still be at the direct modeling stage or need to notate parts of the problem as they think through it. As children become more adept, they can and should be challenged to do appropriate computations mentally.

Today's mathematics curriculum and instruction must focus on preparing students to be mathematically proficient and compute accurately, efficiently and flexibly. If our goal is to create students who meet the above criteria, we must provide opportunities for them to grapple with number relationships, apply these relationships to computation strategies, and discuss and analyze their reasoning. Number Talks are an excellent way to help our students. Here is an example of a 2nd Grade Number Talk. As you watch the video, consider the following questions.

Video: Second Grade Number Talk

Reflection

- How does the teacher record each strategy to provide access for the class?
- Which strategies are easiest for you to understand?
- Which strategies are more challenging to follow?
- What mathematical concepts are being built upon during the number talk?
- How did the teacher bring these ideas to the forefront of her class?

Participant Activity

Solve the following problem using mental math. Be prepared to show your strategy to the whole group.

342 + 153 + 481

Invented Strategies for Addition and Subtraction

Children should be able to use strategies for addition and subtraction that they understand and can use efficiently. Your goal might be that each of your children has at least one or two methods that are reasonably efficient, mathematically correct, and useful with lots of different numbers. Expect different children to settle on different strategies that play to their strengths.

Children do not spontaneously invent wonderful computational methods while the teacher sits back and watches. In various experimental programs, children tended to develop or gravitate toward different strategies, suggesting that teachers and the programs do have an effect on the methods children develop.

Vershcaffel et al., 2007

Creating a Supportive Environment

Children who are attempting to investigate new ideas in mathematics need a safe and nurturing classroom environment in which they can take risks, test conjectures, and try new approaches.

At your table, generate a list of the ways you create a supportive environment. Are there things you specifically avoid? Things you purposefully do? These will be shared and charted together.

Ideas for Creating a Supportive Environment

- Avoid immediately identifying the right answer when a child states it. Give other children a chance to consider whether they think it is correct.
- Expect and encourage student-to-student interactions, questions, and discussions.
- Promote curiosity and openness to new ideas and trying new things.
- Talk about both right and wrong ideas in non-evaluative and non-threating ways.
- Move less sophisticated thinking to more sophisticated thinking through encouragement, coaching, or strategic questioning (Assessing and Advancing Questions).
- Use familiar context and story problems to build a background and connect to children's experiences.

Models to Support Invented Strategies

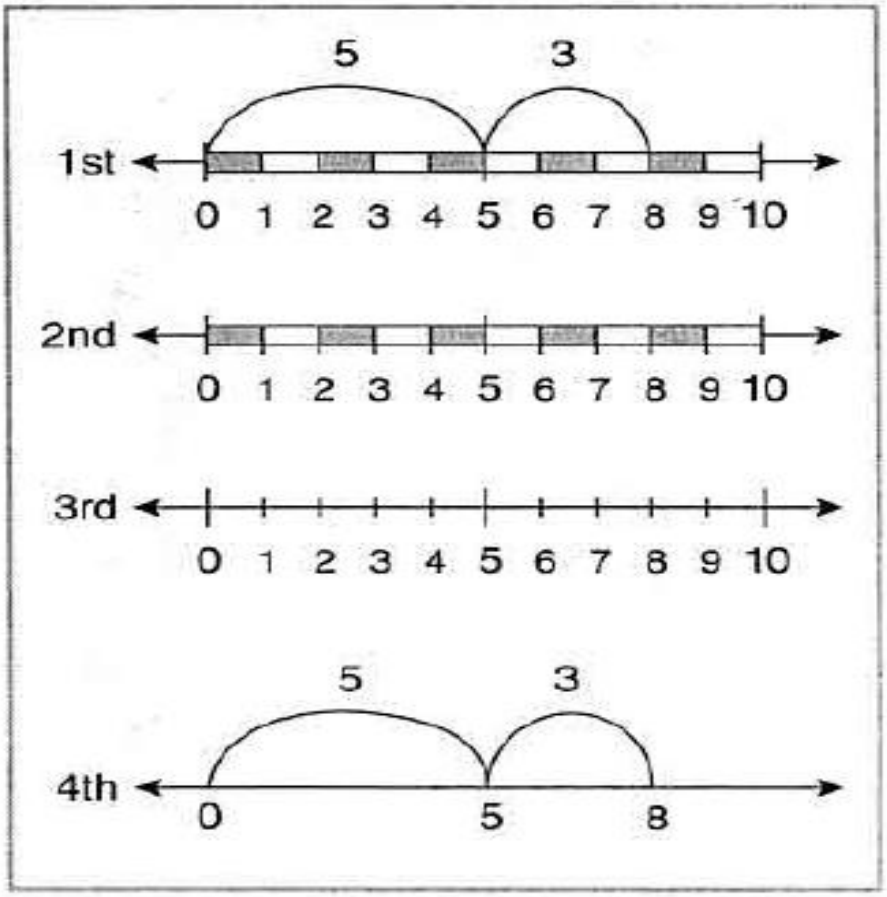
There are three common types of invented-strategy models that children come up with to solve addition and subtraction situations:

Strategy	Model
Split Strategy (Decomposition)	Arrows or lines to explicitly indicate how two computations are joined together.
Jump Strategy (Counting on or counting back)	Empty number line has been found to be more flexible and children are less prone to making computational errors when using it.
Shortcut Strategy (Compensation)	Because this strategy involves the flexible adjustment of numbers, you may find simply recording with an equation or an open number line to be most effective.

Number Lines

An innovative approach to computation is the use of open number lines to demonstrate addition and subtraction. A number line measures distances from zero the same way a ruler does, and can sometimes present conceptual difficulties to younger children, as they can mistakenly focus on the hash marks or numerals on a number line instead of the spaces (units of length) when counting.

Van de Walle provides a helpful visual of the sequence in which we should present number lines to make them meaningful for children.



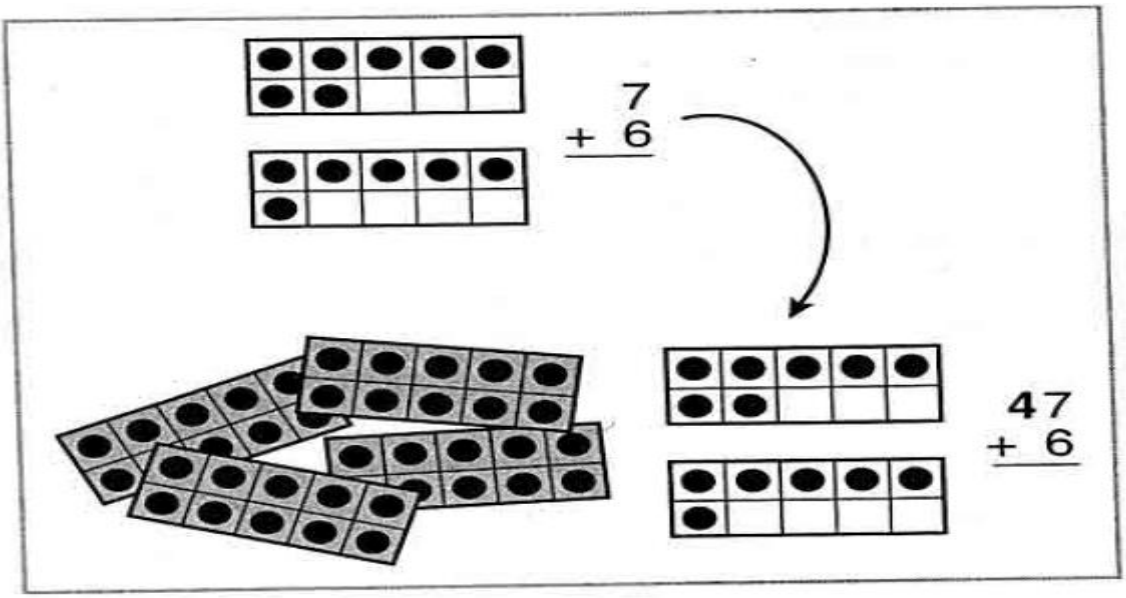
Reflection

- What are the subtle differences you notice in each of these number lines?
- Do you feel this is an effective way for children to communicate their reasoning? Why or why not?

Adding and Subtracting Single Digits

When adding or subtracting small amounts or finding the difference between two reasonably close numbers, many children will use counting to solve the problem. One goal should be to extend children's knowledge of basic facts and the ten structure of the number system so that counting is not required.

Students can use the Up Over 10 strategy when adding numbers that cross from one decade to another. For example, the computation 58+6 can we thought of as 58+2+4. Here, students add on to get 10, recognizing that 58+2=60, and then add the remaining singles, 60+4=64. Similarly, for subtraction, students can use the Down Over 10 strategy. For example, 53-7: take away 3 to get 50, then subtract four more to get 46.



Van de Walle, Lovin, Karp, & Bay-Williams., 2014

Adding and Subtracting Tens and Hundreds

As you move children from single-digit to two-digit numbers, adding and subtracting tens and hundreds is an important transition. Sums and differences involving multiples of 10 or 100 are easily computed mentally. Write a problem such as the following on the board:

300 + 500 + 20

Challenge children to solve it mentally. Ask them to share how they did it. Listen for the use of place value words: "3 hundred and 5 hundred is 8 hundred and 20 is 820." Early problems should not require regrouping. Then, move to more difficult problems that do require regrouping. Using base ten models can help children think in terms of units of tens and hundreds.

Adding Two-Digit Numbers

Problems involving the sum of two two-digit numbers will usually produce a wide variety of strategies. Some of these strategies will involve starting with one or the other number and working from that point, either by adding on to get to the next ten or by adding tens to get from one number to the other.

Consider the following problem:

Two Scout troops went on a field trip. There were 46 Girl Scouts and 38 Boy Scouts. How many Scouts in all went on the trip?

Chart Activity: Let's consider four different strategies. Think about what it sounds like and how you would model it.

Strategy	What does it sound like?	Model
Add Tens, Add Ones, then Combine		
Add on Tens, then Add Ones		
Move Some to Make Tens		
Use a Nice Number and Compensate		

Participant Activity

Try using addition to solve this problem in as many different ways as you can. How many ways are like those we just discussed? Can you think of another way?

367 + 155

Your goal is to help children develop strategies that are efficient and that make sense to them. Listening to how some of their classmates use composition or decomposition can help others be more aware of different ways to adjust numbers.

Subtracting by Counting Up

This is a powerful way to subtract. Children working on the think-addition strategy for their basic facts can also be solving problems with larger numbers. The concept is the same.

Using a join with change unknown or missing part problem will encourage the counting up strategy.

Consider the following problem:

> Sam had 46 baseball cards. He went to a card show and got some more cards for his collection. Now he has 73 cards. How many cards did Sam buy at the card show?

Chart Activity: Let's consider three different strategies. Think about what it sounds like and how you would model it.

Strategy	What does it sound like?	Model
Add Tens, Add Ones, then Combine		
Add Tens to Overshoot, Then come back		
Add Ones to Make a Ten, then Tens and Ones		

Take Away Subtraction

Using take away subtraction is more difficult when done mentally. However, take away strategies are common, probably because many textbooks emphasize take away as the meaning of subtraction.

Consider the following problem:

> There were 73 children on the playground. 46 second grade students left the playground. How many children were still outside?

Chart Activity: Let's consider five different strategies. What does it sound like and how you would model it?

Strategy	What does it sound like?	Model
Take tens from the tens, then subtract the ones		
Take tens from the tens, then subtract the ones		
Take away tens, then ones		
Take extra tens, then add back		
Add to the whole if necessary		

KEY IDEA #9

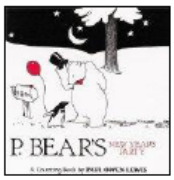
Flexible methods for computation require a strong understanding of the operations and properties of the operations, especially the commutative and associative property. Understanding how addition and subtraction are related as inverse operations is also important.

Literature Connection:

P. Bear's New Year's Party: A Counting Book by Paul Owen Lewis

A dapper polar bear has an elegant New Year's party and invites all of his animal friends—one whale, two horses, three cows, and so forth until midnight. As each party animal arrives, children learn to count. The following task can be used.

P. Bear's New Year's Party



Materials: copy of *P. Bear's New Year's Party* by Paul Owen Lewis

1. After listening to the story solve the following problem:

   How many guests came to P. Bear's party?

2. Use pictures, numbers or words to show your thinking.

Retrieved from www.k5mathteachingresources.com

Troubleshooting

Participant Activity

Some children may become confused when they hear their classmates talking about adding back numbers. For example, if they are solving 83-29, a child may say they took away 30 then added 1 back. Those students who are confused usually do not understand why they are adding back 1. They think that because you added 1 to 29 to make it 30 you should subtract 1 from the answer. Discuss with your group how you would help this student understand this strategy.

Student Activity: HOW FAR TO MY NUMBER?

-Battista, M., 2012

1. Children work in pairs for this activity.
2. Without communicating to each other, one child writes down a number less than 50 while the other child writes down a number greater than 50. You may choose to limit the size of the second number depending on the children.
3. The children work together to find out how much more must be added to the smaller number to get to the larger number.
4. Once an answer is determined, they should use little ten frame cards to represent the smaller number and the amount they found to see whether the total matches the larger number.

Another idea to help children think about using an adding–on approach is to show a number such as 28 with little ten frame cards and ask, “What goes with 28 to make 53?” You can do the same with three-digit numbers with or without the use of models.

Putting it all together with a Task Arc

A task arc is a set of related lessons which consist of several tasks and their associated lesson guides. The Lesson Progression Chart found in each task arc outlines the growing focus of content to be studied and the strategies and representations students may use. The lessons are sequenced in deliberate and intentional ways and are designed to be implemented in their entirety. It is possible for students to develop a deep understanding of concepts because a small number of standards are targeted. Lesson concepts remain the same as the lessons progress, however the context or representations change.

Note:

There are four task arcs for first grade. They can be found here: http://tncore.org/math/instructional_resources/grades/grade1.aspx.

There are three task arcs for second grade. They can be found here: http://tncore.org/math/instructional_resources/grades/grade2.aspx

Participant Activity

With a partner or small group, review the lesson progression chart in the task arc for your grade level. Record your “noticings” and “wonderings” below.

TN Department of Education

Grade 1 Task Arc

The Relationship between Addition and Subtraction

http://www.edutoolbox.org/system/files/rasp_file/1stGradeTaskArc2.pdf

	Task 1 **Apples and Oranges** *Developing Understanding*	**Task 2** **Colored Beads** *Developing Understanding*	**Task 3** **Playground** *Developing Understanding*	**Task 4** **Pet Store** *Developing Understanding*
Content	• Solves "take from" situational problems, result (difference) unknown • Introduces subtraction as an unknown-addend problem • Utilizes part-part-whole relationship	• Solves "take from" situational problems, result (difference) unknown • Explores subtraction as an unknown-addend problem • Utilizes part-part-whole relationship	• Solves "take from" situational problems, result (difference) unknown, change unknown • Explores subtraction as an unknown-addend problem • Utilizes part-part-whole relationship	• Solves "take from" situational problems, result (difference) unknown, change unknown • Establishes subtraction as an unknown-addend problem • Utilizes part-part-whole relationship
Strategy	• Counting Back from Whole • Counting On from Part • Math Facts, doubles	• Counting Back from Whole • Counting On from Part • Math Facts, doubles	• Counting Back from Whole • Counting On from Part • Math Facts, doubles +1	• Counting Back from Whole • Counting On from Part • Subtraction as undoing of Addition
Representations	• Starts with context that requires students to make sense of a situation • Asks students to represent the situation on the part-part-whole mapping device and to write equations	• Starts with context that requires students to make sense of information in a table to solve the problem • Asks students to represent the situation on the part-part-whole mapping device and to write equations	• Starts with context that requires students to make sense of a situation • Asks students to represent the situation on the part-part-whole mapping device and write equations	• Starts with context that requires students to make sense of information in a table to solve the problem • Asks students to represent the situation on the part-part-whole mapping device and to write equations

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	Task 5 Subtraction as Addition *Solidifying Understanding*	Task 6 Bowls of Marbles *Developing Understanding*	Task 7 Jackson's Blocks *Developing Understanding*	Task 8 Sets of Equations *Solidifying Understanding*
Content	• Solidifies subtraction as an unknown-addend problem • Utilizes part-part-whole relationship	• Solves "take apart" situational problems, addend unknown • Applies concept of subtraction as an unknown-addend problem • Utilizes part-part-whole relationship	• Interprets "take apart" context • Utilizes part-part-whole relationship	• Applies understanding of part-part-whole relationship
Strategy	• Previously listed strategies	• Previously listed strategies	• Previously listed strategies	• Previously listed strategies
Representations	• Starts with a subtraction equation and a related unknown-addend equation • Asks students to use a subtraction equation to write unknown-addend equations and solve equations	• Starts with context that requires students to make sense of information in a table to solve the problem • Asks students to represent situation on the part-part-whole mapping device and to write related addition and subtraction equations	• Starts with a visual representation and challenges students to think flexibly about the quantities represented as they impose the meaning of the part-part-whole relationship • Asks students to write related addition and subtraction equations	• Starts with numbers, related equations, and a visual model and challenges students to think flexibly about the quantities and the way the quantities are represented • Asks students to represent sets of numbers with a diagram or to reference part-part-whole relationship and write related addition and subtraction equations • Repeated reasoning expected

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TN Department of Education

Grade 2 Task Arc

Addition: Flexible Thinking and Problem-Solving Strategies

http://www.edutoolbox.org/system/files/rasp_file/2ndGradeTaskArc-Addition-Flexiblethinking.pdf

	Task 1 **Prizes for the Class** *Developing Understanding*	**Task 2** **Collections** *Developing Understanding*	**Task 3** **What's Changing?** *Solidifying Understanding*	**Task 4** **Sit-Ups** *Developing Understanding*
Content	Explores place value, adding 10 and groups of 10 and adding numbers according to their place value by solving "put together" situational problems, whole unknown.	Establishes understanding of place value, adding 10 and groups of 10 and adding numbers according to their place value by solving "put together" situational problems, whole unknown.	Solidifies understanding of place value, adding 10 and groups of 10 by solving addition equations, sum unknown. Recognizes that only the tens place changes when adding 10.	Solves "put together" situational problems, whole unknown. Explores conservation, moving amounts between addends without changing the sum by solving "put together" situational problems, whole unknown.
Strategy	Count On by Ten from a two-digit number. Add by Place Value to combine like amounts.	Count On by Ten from a two-digit number. Add by Place Value to combine like amounts.	Count On by Ten from a two-digit number. Add by Place Value to combine like amounts.	Count On by Ten from a two-digit number. Add by Place Value to combine like amounts. Juggle Amounts to Make Friendly Numbers (moves amounts from one addend to another and adds).
Representations	Starts with context. Asks students to use the part-part-whole map to construct a model or to use a hundreds chart to represent a situation and write equations.	Starts with context. Asks students to use the part-part-whole map to construct a model or to use a hundreds chart to represent a situation and write equations.	Starts with a series of equations and asks students to solve for the sum. Asks students to analyze sets of equations and to determine how the sums change. Asks students to use the part-part-whole map to construct a model or to use a hundreds chart to represent addition equations.	Starts with context presented in the form of a table. Asks students to use the part-part-whole map to construct a model or to use a hundreds chart to represent a situation and write equations.

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	Task 5 **Flowers** *Developing Understanding*	**Task 6** **Sports Equipment** *Developing Understanding*	**Task 7** **Fieldtrip to the Zoo** *Developing Understanding*	**Task 8** **Flexible Thinking** *Solidifying Understanding*
Content	Solves "put together" situational problems, whole unknown. Explores conservation.	Solves "put together" situational problems, whole unknown. Establishes understanding of conservation. Explores compensation (rounding up and adjusting for increase).	Solves "put together" situational problems, whole unknown. Establishes understanding of compensation.	Solidifies understanding of compensation, conservation, and place value, by solving addition equations, sums unknown.
Strategy	Count On by Ten from a two-digit number. Add by Place Value to combine like amounts. Juggle Amounts to Make Friendly Numbers. Make a Ten from Ones (uses the ones to make a ten and then adds).	Count On by Ten from a two-digit number. Add by Place Value to combine like amounts. Juggle Amounts to Make Friendly Numbers. Make a Ten from Ones. Make Friendly Numbers and Adjust (increases an addend, adds and then adjusts the resulting sum).	Count On by Ten from a two-digit number. Add by Place Value to combine like amounts. Juggle Amounts to Make Friendly Numbers. Make a Ten from Ones. Make Friendly Numbers and Adjust.	Count On by Ten from a two-digit number. Add by Place Value to combine like amounts. Juggle Amounts to Make Friendly Numbers. Make a Ten from Ones. Make Friendly Numbers and Adjust.
Representations	Starts with context presented in the form of a table. Asks students to use the part-part-whole map to construct a model or to use a hundreds chart to represent a situation and write equations.	Starts with context presented in the form of a table. Asks students to use the part-part-whole map to construct a model or to use a hundreds chart to represent a situation and write equations.	Starts with context presented in the form of a table. Asks students to complete the table by filling in the missing sums. Asks students to use the part-part-whole map to construct a model or to use a hundreds chart to represent a situation and write equations.	Asks students to analyze sets of expressions, determine what changed in the sets, and then apply the same reasoning to another set of expressions.

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Closing Activity

Think back to the areas we have explored concerning Addition and Subtraction Concepts:

- Connecting Place Value to Addition & Subtraction
- Computation Strategies
    - Direct Modeling
    - Invented Strategies
    - Mental Computation

Although we have looked at a variety of activities, this is not an exhaustive list.

Reflection

- What other activities do you use in your classroom to promote Addition and Subtraction Concepts?

Discuss these strategies in your small group. Be prepared to share whole group. Record any ideas that you would like to incorporate in your classroom in the following pages.

Collaborative Notes/Activities – Addition & Subtraction

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