PBS Lesson Series

Math: Grade 8, Lesson 10, Using Functions to Model Linear Relationship

Lesson Focus: Using Functions to Model Linear Relationships
Practice Focus: Students will focus on practicing writing equations and identifying input and output values in order to model linear functions.
Objective:

- Refine strategies for using functions to model linear relationships
- Refine understanding of the meaning and interpretation of rate of change and initial value in a variety of contexts

Key Vocabulary:

- Initial value: In a linear function, the value of the output when the input is 0
- Linear function: A function that can be represented by a linear equation. The graph of a linear function is a nonvertical straight line.
- Quadrants: The four regions of the coordinate plane that are formed when the x-axis and y-axis intersect at the origin
- Rate of change: In a linear relationship between x and y, it tells how much y changes when x changes by 1
- Slope: For any two points on a line, the $\frac{rise}{run}$ or $\frac{change\ in\ y}{change\ in\ x}$. It is a measure of the steepness of a line. It is also called the rate of change of a linear function.
- Slope-intercept form: A linear equation in the form y = mx + b, where m is the slope and b is the y-intercept
- Y-intercept: The y-coordinate of the point where a line, or a graph of a function, intersects the y-axis

TN Standards: 8.F.B.4, 8.F.B.5
Teacher Materials:

- Whiteboard and Markers, Graph Paper if available
- Student Practice Packet

Student Materials:

- Paper and a pencil, and a surface to write on
- Calculator not required but may be used to check calculations.
- Optional but helpful: Graph Paper

Note: There are a charts and graphs that will need to be prepared ahead of time to show to students during the lessons this week.

Teacher Do	Student Do
Opening (1 min) **Hello! Welcome to Tennessee's At Home Learning Series for math! Today's lesson is for all our 8th graders out there, though all students are welcome to tune in. This lesson is the tenth in our series.** **My name is ____ and I'm a ____ grade teacher in Tennessee schools! I'm so excited to be your teacher for this lesson! Welcome to my virtual classroom!**	Students get materials ready for the lesson.

If you didn't see our previous lesson, you can find it on the TN Department of Education's website at www.tn.gov/education. If you don't already have the student packet for this lesson, you can also find it online at www.tn.gov/education. You can still tune in to today's lesson if you haven't see any of our others. But, it might be more fun if you first go back and watch our other lessons since we'll be talking about things we learned previously. **Today we will be learning Using Functions to Model Linear Relationships in mathematics! Before we get started, to participate fully in our lesson today, you will need:** • **Paper and a pencil, and a surface to write on** • **A calculator is not required but may be used to check calculations.** • **Optional but helpful material would be graph paper** **Ok, let's begin!**	
<u>Intro</u> (2 min) **Welcome to the tenth lesson in this series, and if you completed all nine previous lessons, congratulations! That was a big accomplishment!** **This lesson focuses on modeling linear relationships, but our first problem will use Fahrenheit and Celsius. The Celsius scale was introduced in the mid-1700s by Andres Celsius, and is a metric measure of temperature. The Fahrenheit scale was introduced in 1724 by Daniel Gabriel Fahrenheit. In the United States, temperatures are usually reported in degrees Fahrenheit, but most other countries use degrees Celsius. The only value for which the temperatures are the same is 240°.** **Are you ready to look at the first problem?** [Pause] **OK, let's get started!**	Students are thinking about this background on Fahrenheit and Celsius and recalling what they should know about modeling linear relationships.
<u>Teacher Model</u> (10-12 min) <u>Objective 1:</u> <u>using functions to model linear relationships</u> **You've learned in science class that Celsius and Fahrenheit are two different scales for measuring temperature. The freezing point of water is 0° C, or 32° F** [write or show on board]. **The boiling point of water is 100° C, or 212° F.** [Write or show on board]	Objective 1: Students may identify the initial value by noticing that the freezing point of water is 0°C or 32°F.

Let's think about how we would write an equation that shows the temperature in degrees Fahrenheit as a function of the temperature in degrees Celsius. Are the Celsius temperatures the input values, or the output values?
[Pause]

To write our equation, we would use (0, 32) and (100, 212). [Write or show] **The initial value is 32. Here's what the rate of change would look like:** [write or show]

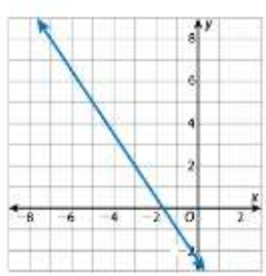
$$\frac{\mathbf{212-32}}{\mathbf{100-0}}$$
$$=\frac{\mathbf{180}}{\mathbf{100}}$$
$$=\frac{\mathbf{9}}{\mathbf{5}}$$

So, our equation would look like this:

$$\boldsymbol{y=\frac{9}{5}x+32}$$

<u>Objective 2 Write an equation for a linear function from a graph, from two points, or from a verbal description.</u>

Let's take a look at this graph [show graph]. **What is the equation of the function show by the graph?**



To plot your line on the graph, use (-5, 5) and (-1, -1).

$$\boldsymbol{m}=\frac{\mathbf{5-(-1)}}{\mathbf{-5-(-1)}} \qquad \boldsymbol{y=mx+b}$$
$$=\frac{\mathbf{6}}{\mathbf{-4}} \qquad \mathbf{-1}=-\frac{\mathbf{3}}{\mathbf{2}}\,\mathbf{(-1)}+\boldsymbol{b}$$
$$=-\frac{\mathbf{3}}{\mathbf{2}} \qquad \mathbf{-1}=\frac{\mathbf{3}}{\mathbf{2}}+\boldsymbol{b}$$
$$-\frac{\mathbf{5}}{\mathbf{2}}=\boldsymbol{b}$$

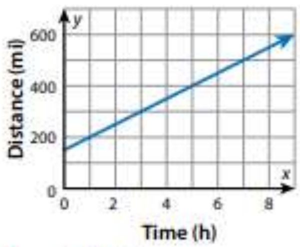
The solution is:

Objective 2: Students may check their equations by substituting a value for x, and then simplifying the right side of their equation to find a corresponding value for y. Then, they may check that the ordered pair (x, y) is on the line shown in the graph

$$y - -\frac{3}{2}x - \frac{5}{2}$$ <u>Objective 3 understanding of the meaning and interpretation of rate of change and initial value in a variety of contexts</u> **Lillie uses some birthday money to open a bank account. Then she deposits the same amount into the account each week. The equation y = 35x + 100 represents the total amount in the account after x weeks. What does the initial value of the function represent? Look at these possible answers – which do you think it is?** [Ask students to show or write a reason for their response, and give time to do so.] **a. The amount deposited each week** **b. The amount used to open the bank account** **c. The total amount in the account after the first week of saving** **d. The total amount in the account after x weeks** **Think about how much Lillie uses to open the bank account, and how much she deposits each week. Which answer do you think is correct?** [Pause] **Is it A? That's a good guess, but let's look again at the initial value of the function with the rate of change. The initial value is the amount in the account after 0 weeks, which is the amount used to open the account. This would mean that B is the correct answer, because answer A is the rate of change of the function. Answer C is the value of y when x = 1, and answer D is the value of y after x weeks.**	Objective 3: Students will examine the answer choices in relation to the context of the problem to choose a response and provide a reason for their response.
<u>Guided Practice</u> (10-12 min) **Now, let's look at finding the rate of change and initial value of a linear function using its graph. If you have some graphing paper, you can use it for this next problem.** **Let's plot some points at (4,5) and (8,7), then draw a line through them.** [Plot points on four-quadrant coordinate plane] **How can you use the graph to find the slope of this line?** [Pause] **That's right, you will divide the rise by the run. Let's mark this together.** [Write and label the rise and run]	Students use equations to model linear functions by finding the equation of a line using the features of a graph.

Let's calculate the slope. What is the slope, and how does the slope of the line relate to the rate of change for the related linear function? [Pause] **It is $\frac{1}{2}$, and it has the same value.** **Now, how can you use the slope of the line to help identify the y-intercept?** [Write or show on the board] **You will start at (4, 5) and then move 1 unit down and 2 units left until you reach the y-axis at (0, 3).** **What is the y-intercept?** [Pause] **That's right, it's 3. How does the y-intercept of the line relate to the initial value for the related linear function?** [Pause] **It has the same value.** **What is the equation for the linear function modeled by the line**? [Pause] **It will look like this:** [Write or show on board] $$\boldsymbol{y = \frac{1}{2}x + 3}$$ **What are some real-life functions that this equation could represent? For example, a spider starts at a height of 3 meters off the ground. It climbs up a wall 1 meter every 2 minutes. The function** [show on board] **gives its height in meters, y, as a function of the number of minutes, x. Can you think of any other examples? Go ahead and write it down!** [Pause] <u>Additional Practice if needed:</u> **Let's think through a couple more. Here's a shorter one first. Let's just think this through:** [teacher write/show and read aloud] **"The equation y = 0.15x + 0.40 represents the cost of mailing a letter weighing one ounce or more. "** **In the equation, which part of that description is represented by *x*?** [pause] **If you said that *x* represents the weight of the letter, then you are right! Now what would *y* represent?** [pause] **If you said that *y* represents the total cost of mailing the letter, then you're right again!** **Now, Using the phrase *is a function of*, how would you state the scenario? Think about it for a quick second.** [pause] **If you said that "the cost of mailing a letter is a function of the weight of the letter", then you are on target! I think you're getting the hang of this!**	

Now, let's review one more problem. This time we will use a distance and time graph. Let's take a look. See what you can infer from this graph: [teacher show the graph]



See if you can identify the following things in the graph:

- **The initial value or y-intercept**
- **At least two ordered pairs on the graph**
- **The rate of change on the graph.**

[pause – a little longer than other pauses]

Okay, now I'm going to point to some of these things on the graph. Check your thinking with mine.
[point to the location that is approximately (0,150)]
This position on the graph is the y-intercept or initial value. Visually estimating, that looks like it would be at about 150.
[point to the location (3,300)]
There are several individual sets of ordered pairs that we can identify on this graph. This is one of them at (3, 300).
[point to the location (7, 500)]
This is another one at (7,500).
Did you see others you could use? [pause]
I did too! I saw these [teacher point to each location as they are stated) **at (1,200) and (3, 300) and (5, 400). Remember that any two of these ordered pairs can be used to help you find the rate of change by calculation. You could also use the graph to visualize the rise over the run.**

Now, let's see if we can create the equation that models this distance and time graph. We should have enough information to use everything we've learned this week. I'm going to give you a few minutes to start working. Remember, you need to find the rate of change! [longer pause]

Now, this is the final equation that I came up with. [teacher write/show and read aloud]
Y = 50x + 150
Did you get the same? [pause]

If you didn't, then let's go back and check your arithmetic. If you did, then great! I'll walk through the steps now. [teacher write/show and speak aloud] **Let's start with finding the rate of change or slope. I'll use the ordered pairs that I first saw at (3,300) and (7,500) to calculate the rate of change. Remember that slope is the change in the y values (rise) over the change in the x value (run).** **Slope =** $\frac{500-300}{7-3} = \frac{200}{4} = 50$ **Now, we can use one point and the rate of change to find the exact initial value. We think it is at 150, but we want to be sure since graphs are sometimes more difficult to read for individual ordered pairs.** **Starting with y = mx + b, substitute in the rate of change and one ordered pair. Remember, you can use any one of the ordered pairs that you identified previously. It should not make any difference in the final result because they are all on the same line.** **Y = mx + b** **I'll use the ordered pair (1, 200) with the rate of change of 50.** **By substitution** **200 = 50(1) + b** **200 = 50 + b** **200 – 50 = 50 – 50 + b** **150 = b** **So, the initial value is what we thought it was at 150.** **Therefore, the final equation that models this distance time graph is y = 50x + 150.** **You really are getting this! Keep going with some independent practice and review from the whole week.**	
<u>Independent Practice</u> (1 min) **Terrific work today, students! Today, we explored the Using Functions to Model Linear Relationships in mathematics. I hope you are making connections between the linear equation and the graphic representation of the functions we are modeling. After this lesson, you will have a few problems to practice on your own. I will show you the independent practice problems now, or you can find them in the student**	

practice for this lesson posted on our website, <u>www.tn.gov/education</u>. [Teacher shows student practice page under document camera or camera zooms in on student practice page.] **Good luck and do your best!**	
<u>Closing</u> (1 min) **I enjoyed reviewing Using Functions to Model Linear Relationships in mathematics with you! Thank you for inviting me into your home. I look forward to seeing you in our next lesson in Tennessee's At Home Learning Series! Bye!**	

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