Math: Grade 8, Lesson 9, Using Functions to Model Linear Relationship

Lesson Focus: Using Functions to Model Linear Relationships
Practice Focus: Write an equation for a linear function from a verbal description
Objective:
- Develop strategies for writing an equation for a linear function when given a verbal description of the situation.
- Recognize that a rate of change and initial value are needed to write the equation of a linear function.

Key Vocabulary:
- Initial value: In a linear function, the value of the output when the input is 0
- Linear function: A function that can be represented by a linear equation
- Rate of change: In a linear relationship between x and y, it tells how much y changes when x changes by 1

TN Standards: 8.F.B.4, 8.F.B.5
Teacher Materials:
- Whiteboard and Markers, Graph Paper if available
- Student Practice Packet

Student Materials:
- Paper and a pencil, and a surface to write on
- Calculator not required but may be used to check calculations.
- Optional but helpful: Graph Paper, Straightedge, Tracing Paper

Teacher Do	Student Do
Opening (1 min) **Hello! Welcome to Tennessee's At Home Learning Series for math! Today's lesson is for all our 8ᵗʰ graders out there, though all students are welcome to tune in. This lesson is the ninth in our series.** **My name is _____ and I'm a _____ grade teacher in Tennessee schools! I'm so excited to be your teacher for this lesson! Welcome to my virtual classroom!** **If you didn't see our previous lesson, you can find it on the TN Department of Education's website at www.tn.gov/education. If you don't already have the student packet for this lesson, you can find it online at www.tn.gov/education. You can still tune in to today's lesson if you haven't see any of our others. But, it might be more fun if you first go back and watch our other lessons since we'll be talking about things we learned previously.**	Students get materials ready for the lesson.

Today we will be learning Using Functions to Model Linear Relationships in mathematics! Before we get started, to participate fully in our lesson today, you will need:
- **Paper and a pencil, and a surface to write on**
- **Calculator is not required but may be used to check calculations.**
- **Optional but helpful materials are Graph Paper, Straightedges, and Tracing Paper**

Ok, let's begin!

Intro (2 min)

There are many instances where a problem is not presented as an equation at all. The problem is instead presented as a verbal description, and you have to make sense of the problem in order to find the answer to the question.

One strategy for making sense of the problem is to use Three Reads – in other words, read the problem aloud a first time, then read it again, looking for cues that will help you understand what the problem is asking. Finally, read the problem a third time to determine the important quantities in the description that will be needed to solve the problem.

We'll be walking through a few verbal descriptions to analyze them for the pertinent information that will be used to solve the problem.

Are you ready?
 [Pause]

Good! Let's get started!

Teacher Model (10-12 min)

Objective 1: Develop strategies for writing an equation for a linear function when given a verbal description of the situation.

If you have them, get your graph paper, straightedge, or tracing paper, and let's dive in!

[Read problem aloud]
Kadeem spends the afternoon reading a book he started yesterday. He reads 120 pages in 3 hours. One hour after Kadeem begins reading, he is on page 80. Write an equation for the page he is on, y, as a function of minutes spend

Objective 1:
Students begin by reading the verbal description aloud and examining the scenario to explain what the problem is asking. Then students determine the important quantities in the description that will be needed to solve the problem.

reading, x. What page number was he on when he started reading today?

If you'd like, pause the video and read the problem again, either aloud or to yourself.
[Pause]

Ready? [Pause]
Let's take a close look at the information provided. You can graph the function. Let's plot the point (60, 80) on a graph.
[show on the graph]

From here, you can find the rate of change, which is the slope of the line.
If Kadeem started the book at 0 minutes, he would be on page 120 after 180 minutes. How many hours is that? [Pause]
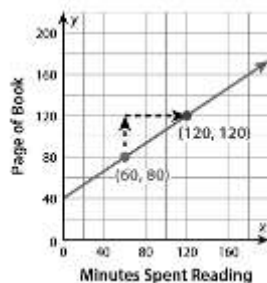
Did you say three hours? [Pause]

Let's take a look.
This is your slope:
Slope = $\frac{120}{180} = \frac{40}{60} = \frac{2}{3}$

Use the slope to plot another point at (120, 120). Draw a line through the points and identify the y-intercept. It should look like this: [show on the whiteboard]

Objective 2: Recognize that a rate of change and initial value are needed to write the equation of a linear function.

What are some different ways of finding the initial value?
You can calculate the rate of change and initial value like this:

$\frac{120 \, pages}{3 \, hours}$ **can be written as** $\frac{120 \, pages}{180 \, minutes}$

The rate of change is $\frac{2}{3}$ **page per minute.**

Objective 2:
Students will consider the following questions:
Which quantities in the problem do you need in order to find the rate of change?
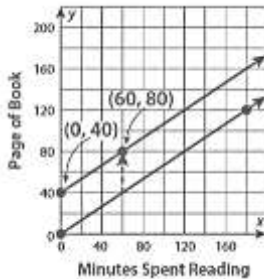Does the description give the initial value?
If he reads 120 pages in 3 hours, how was he able to be on page 80 after the first hour?

To find the initial value, use the equation for a linear function. Then substitute the rate of change and the point (60, 80).

y = mx + b becomes

80 = $\frac{2}{3}$ (60) + b

But (60, 80) is not on this line, so translate up until it is. Keep in mind, each grid space along the x-axis represents 20 minutes, which is the same as $\frac{1}{3}$ hour. [Show on the graph]

The line looks like it passes through (0, 40), so the y-intercept is 40.

Your slope would be: $\frac{80-40}{60-0} = \frac{40}{60} = \frac{2}{3}$

Your equation would be: y = $\frac{2}{3}$ x + 40

This shows Kadeem was on page 40 when he began reading today.

Let's look at it another way.

120 pages in 3 hours is $\frac{120}{3}$ = 40 pages per hour.

[write or show]

If he was on page 80 after one hour, then he must have started on page 40 because 80 − 40 = 40.

120 pages in 180 minutes is $\frac{120}{180}$ = $\frac{2}{3}$ page per minute.

The rate of change is $\frac{2}{3}$ and the initial value is 40.

So the equation is y = $\frac{2}{3}$ x + 40.

<u>Guided Practice </u>(10-12 min) **Let's take a look at a couple of other verbal problems. Think about the strategies we used to solve the last problem, and how they might be useful for these problems as well.** [Read problem aloud and show on whiteboard]	Students will apply the previously learned steps to these practice problems with support guidance from the teacher.

A lawn mower the energy rating shown in the box. A full tank of gas can power the lawn mower for about 6 hours. What equation can be used to find the amount of gas left in the tank, y, as a function of the mowing time, x? [Pause]

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ENERGY RATING Ride-on Lawn Mower

Fuel Capacity          Mowing Time
 8 gallons              6 hours
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As you can see, the initial value is 8 gallons, which is the amount of gas in a full tank. Because the gas is being consumed and its amount is decreasing as the mower is running, the slope would be negative.

So, your rate of change = $\frac{8\ gallons}{6\ hours} = \frac{4}{3}$ gallons per hour

Change in the value of the function refers to the change in y (the output value), and change in the input refers to the change in x.

Your equation would be y = $-\frac{4}{3}$ x + 8

Let's look at one more problem.

Question 2
[Read problem aloud and show on whiteboard]

Aniyah is driving home at a constant speed. After 20 minutes, she is 70 miles from home. After 1 hour, she is 40 miles from home. What equation models her distance from home, y, as a function of time, x? [Pause]

One hour is 60 minutes. On a graph, the points (20, 70) and (60, 40) satisfy the function if using minutes. The two points can be used to find the rate of change.

To find the initial value, the rate of change can be substituted for m and the coordinates of one point for x and y in
y = mx + b, and then the equation can be solved for b.

The rate of change, then, can be shown as:
$$\frac{70-40}{20-60} = \frac{30}{-40} = -\frac{3}{4}$$
$$70 = -\frac{3}{4}(20) + b$$

$$70 = -15 + b$$
$$85 = b$$

The solution for y is: $y = -\frac{3}{4}x + 85$

Additional Problems (if Needed):
Let's think about another example. [teacher read write/show and read aloud]
Tressa is stocking small paint cans on shelves. She stocks 100 cans in 2 hours. One hour after she starts, there are 60 cans on the shelves. Write an equation to model the situation.

Let's think about what we know. [pause]
Since we know we are trying to write an equation in the form y = mx + b, let's determine what our variables are. [pause]
The number of cans is a function of time. So, let y be the number of cans, and x be the time in hours.

What else do we know? Look for information in the problem to tell us the rate of change. [pause]
The problem tells us that she stocks 100 cans in 2 hours. That is a rate of change. Now, how many cans would Tressa stock in one hour?[pause]
Right! 50 cans in 1 hour. So, the rate of change is 50.

What else do we know? [pause]
The problem tells us that after one hour after Tressa starts, there are 60 cans on the shelf. We can use this as an ordered pair (1, 60).

Do you think we have enough information to write the equation now? [pause]
Right! Since we have the rate of change and one ordered pair, we can find the initial value. Let's work through this. Start with y = mx + b and see how far you can get. I'll give you some time. [pause]

Okay! Did you make it all the way to the equation? Did you get something like this? [teacher write/show and speak]
Y = 50x + 10

    Here's how I solved it. Follow along with me. [teacher write/show and speak]

y = mx + b
$y = 50x + b$ **substitute in the rate of change**
$60 = 50(1) + b$ **substitute in the ordered pair (1, 60)**

$60 = 50 + b$ solve for b
$60 - 50 = 50 - 50 + b$
$10 = b$
So, the equation that models this linear function is y = 50x + 10 where the initial value is 10 cans on the shelf and the rate of change is 50 cans per hour as Tressa stocks the shelves.

[Additional Example 2:]
Let's look at one more situation. [teacher write/show and read aloud]
A school club is fundraising by selling boxes of homemade desserts. They buy the ingredients for $40 and then boxes for the desserts for $0.50 each. They sell each dessert box for $5.00.

Let's start modeling this situation by writing and equation for the money they spend, *y*, as a function of the number of dessert boxes they buy, *x*. Remember, we are JUST looking at the money the club is spending. Take a minute to think through it and write down an equation you think fits this scenario. [pause]

So, another reminder that we are looking at the amount of money they SPEND only. Does your equation look like this? [teacher write/show and read aloud]

Y = 0.50x + 40
Great! The initial cost is $40 for the ingredients. [point to the 40 in the equation] **The amount they spend per box for packaging is $0.50.** [point to the 0.50 in the equation]

Now, let's look at the equation that would model the money that they COLLECT for the desserts, *y*, as a function of the number of dessert boxes they sell, *x*. Take a minute to think it through, and write down an equation you think fits this scenario. [pause]

Remember, we are only looking at the amount they collect. Does your equation look like this [teacher write/show and read aloud]

y = 5x or maybe you wrote y = 5x + 0
Great! In this case, if they don't sell any dessert boxes, they don't make any money for the fundraiser. So, the initial value is zero. [point to zero in the second equation]
The amount they collect per box is $5 [point to the 5 in the second equation]

Last question! Let's see if we can combine these two equations into a statement about profit! Remember that profit is the difference between the amount collected and the amount spent on supplies.

Let's walk through this together. You will see a similar problem in the student practice today.

If we let the profit be *y*, then we can state that *y is a function of the difference between the amount collected or 5x and the amount spent or 0.50x + 40.*

Follow along, and we will write this out together: [teacher write/show and read aloud as we go]

Profit = (amount collected) – (amount spent)
Substitute in what we know:
Y = 5x – (0.50x + 40)
Simplify the equation this way:
Y = 5x – 0.50x – 40
Y = 4.50x – 40

There was a little bit of tricky arithmetic in that equation simplification – did you catch it? [point back to the change from +40 to – 40]
Great! Now, this equation models the profit, and we can describe it this way.

The profit or fundraising amount the club gets will be $4.50 per box minus the $40 spent on ingredient supplies!

One more bonus question! How many dessert boxes does the club need to sell in order to actually make a profit?
[pause]
There are several ways to figure that one out, but you should have come up with nine dessert boxes. They have to sell that many to recover the cost of the ingredients.

Terrific! Now, I think we really are ready for some independent practice.

Independent Practice (1 min) **Terrific work today, students! Today, we explored writing an equation for a linear function from a verbal description in mathematics. I hope this has helped you to make connections between graphs, points, verbal descriptions, and equation that model linear functions. After this lesson, you will have a few problems to practice on your own. I will show you the independent practice problems now, or you can find them in the student practice for this lesson posted on our website, www.tn.gov/education. I will show you the independent practice problems now, or you can find them in the student practice for this lesson posted on our website, www.tn.gov/education.** [Teacher shows student practice page under document camera or camera zooms in on student practice page.] **Good luck and do your best!**	
Closing (1 min) **I enjoyed reviewing Using Functions to Model Linear Relationships in mathematics with you! Thank you for inviting me into your home. I look forward to seeing you in our next lesson in Tennessee's At Home Learning Series! Bye!**	