TN Department of Education

Seventh Grade Mathematics

Instructional Focus Documents

Introduction:

The purpose of this document is to provide teachers a resource which contains:

- The Tennessee grade-level mathematics standards
- Evidence of Learning Statements for each standard
- Instructional Focus Statements for each standard

Evidence of Learning Statements:

The evidence of learning statements are guidance to help teachers connect the Tennessee Mathematics Standards with evidence of learning that can be collected through classroom assessments to provide an indication of how students are tracking towards grade-level conceptual understanding of the Tennessee Mathematics Standards. These statements are divided into four levels. These four levels are designed to help connect classroom assessments with the performance levels of our state assessment. The four levels of the state assessment are as follows:

- Level 1: Performance at this level demonstrates that the student has a minimal understanding and has a nominal ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Level 2: Performance at this level demonstrates that the student is approaching understanding and has a partial ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Level 3: Performance at this level demonstrates that the student has a comprehensive understanding and thorough ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Levels 4: Performance at these levels demonstrates that the student has an extensive understanding and expert ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.

The evidence of learning statements are categorized in the same way to provide examples of what a student who has a particular level of conceptual understanding of the Tennessee Mathematics Standards will most likely be able to do in a classroom setting.

Instructional Focus Statements:

Instructional focus statements provide guidance to clarify the types of instruction that will help a student progress along a continuum of learning. These statements are written to provide strong guidance around Tier I, on-grade level instruction. Thus, the instructional focus statements are written for levels 3 and 4.

Ratios and Proportional Relationships (RP)

Standard 7.RP.A.1 (Major Work of the Grade)

Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units.
For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose a unit rate to represent a ratio presented as a complex fraction given a visual model. Choose a unit rate to model situations involving whole number quantities.	Choose a unit rate to model situations involving fractional quantities presented as a non-simplified complex fraction. Write ratios to model situations involving whole number quantities measured in like or different units and use the ratio to determine a unit rate.	Write ratios to model situations involving fractional quantities measured in like or different units and use the ratio to determine a unit rate.	Write ratios that model situations involving fractional quantities measured in like or different units, use the ratio to determine a unit rate, and explain the unit rate in terms of the context of the situation. Create contextual problems that model situations involving fractional quantities measured in like or different units and provide the unit rate solution.

Instructional Focus Statements

Level 3:

The focus of this standard is extending a students' understanding of unit rates developed in grade 6 (6.RP.A.2) to computing unit rates using ratios from fractional quantities known as complex fractions. Students developed an understanding of quotients of fractions in 6.NS.A.1. In grade 7, students will utilize this understanding to compute unit rates from fractional quantities. Students should discover that computing unit rates with fractions is the same concept as unit rates with whole numbers. To enhance the foundational understanding of ratios as division of fractions, students should be encouraged to use visual representations or manipulatives to model the fractional quantities in order to compute a unit rate. Students may have misconceptions when writing a ratio in the form of a complex fraction. It is important for students to understand that the fraction bar is the same as the common division

symbol. Also, students should make connections to visual models to grasp the fractional unit rate. Using the example from the standard, students should utilize a visual representation (number line, bar graph, etc.) to see that if a person walks 1/2 mile every 1/4 hour, results in 4 of the 1/2 miles in a total hour, which is equivalently a unit rate of 2 miles per hour. Students should explain their reasoning about computing unit rates with complex fractions with like and different units, including identifying when errors can be presented.

Level 4:

Students should extend their understanding of computing unit rates associated with ratios of fractions to make connections between visual representations and verbal descriptions. This should be explained by using precise mathematical language. Students should also be well versed in working with like and different units and make sense of contextual problems (MP1) to determine when errors or misconceptions could be made. Additionally, students be able to translate their understanding to create contextual problems that model situations involving fractional quantities measured in like or different units and use the ratio to determine a unit rate.

Standard 7.RP.A.2 (Major Work of the Grade)

Recognize and represent proportional relationships between quantities.

7.RP.A.2b

Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose the constant of proportionality (unit rate) when given a graph.	Choose the constant of proportionality when given a table.	Identify the constant of proportionality (unit rate) from a wide variety of representations of a proportional relationship.	Provide a contextual situation to represent a given a table, graph, or equation. Explain the constant of proportionality (unit rate) in terms of the context.

Instructional Focus Statements

Level 3:

Students should employ their understanding of unit rates to identify the constant of proportionality from a wide variety of tables, graphs, equations, diagrams, and verbal descriptions. It is imperative for students to understand that the constant of proportionality in any representation is one unit to a specified quantity or vice versa. In a graph or table, students should identify that (1, y) and (x, 1) are both unit rates and be able to explain the connection each has to the context as well as the connection to each other. In equations, students should be able to identify the unit rate as the "m" value in the equation in y = mx + b or y = mx form.

Level 4:

Students should extend their understanding of identifying unit rates in multiple representations to explain the connections between the unit rate and the verbal context interchangeably. Additionally, students should make connections to both unit rates of (1,y) and (x,1) to the context as well as the connection to one another using precise mathematical vocabulary. As students solidify their understanding, they should be able to identify and explain unit rates in multiple representations when the unit rate is represented in a form other than one to a quantity by employing their knowledge of proportional relationships and prior knowledge of equivalent fractions and explain their reasoning.

Standard 7.RP.A.3 (Major Work of the Grade)

Use proportional relationships to solve multi-step ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose a proportion that models a simple mathematical or real-world situation involving a ratio or percent.	Solve simple ratio or percent problems without a context.	Solve multi-step ratio and percent problems.	Solve multi-step ratio and percent problems in complex situations and explain the solution in terms of the context.

Instructional Focus Statements

Level 3:

Students should extend their prior knowledge of equivalent ratios to solve multi-step ratio and percent problems. Students should have an in-depth, conceptual understanding of generating equivalent ratios and what that means with respect to the contextual problem as they develop an understanding of solving proportions. As students solidify their understanding, other strategies can be introduced for students to efficiently work with numbers that do not lend themselves easily to mental arithmetic. Using cross multiplication to solve problems involving proportional relationships does not lend itself to building a strong conceptual understanding, but can be used as a strategy to work with a larger range of numbers after conceptual understanding is developed.

Students should understand the precise mathematical vocabulary within the examples of the standard in contextual situations and be able to explain how they are used in each situation. As students work with percentages, they should understand that problems involving percent increase or percent decrease require careful attention to the referent whole. Percentages can also be used in making comparisons between two quantities. Students must pay close attention to the wording of such problems to determine what the whole or 100% amount a percentage refers to.

Level 4:

As students extend their understanding of multi-step ratio and percent problems, they should be able to extract essential information from more complex contextual problems. Students should also be able to determine efficient solutions paths to solve ratio and percent problems and explain why they have

chosen the selected solution path. Additionally, students should be able to use precise mathematical vocabulary in their explanations and its connection to the contextual situation.

The Number System (NS)

Standard 7.NS.A.1 (Major Work of the Grade)
Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
7.NS.A.1a Describe situations in which opposite quantities combine to make 0.
7.NS.A.1b Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
7.NS.A.1c Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
7.NS.A.1d Apply properties of operations as strategies to add and subtract rational numbers.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify that the sum of a number and its opposite is 0.	Identify situations in which opposite quantities combine to make 0. Choose a number line diagram that represents a given addition or subtraction problem of rational numbers. Choose a real-world context to represent a given sum or difference of rational numbers.	Generate a number line diagram that represents a given addition or subtraction problem of rational numbers. Add and subtract rational numbers. Explain that the sum $p + q$ is located a distance $\|q\|$ from p and when it goes in the negative direction and when it goes in the positive direction. Create a real-world context to represent a given sum or difference of rational numbers.	Represent addition and subtraction of rational numbers on a horizontal or vertical number line. Provide a model and an explanation to show why $p - q$ and $p + (-q)$ are equal. Create contextual problems that involve applying properties of operations to add and subtract rational numbers and interpret the respective sum or difference.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		Identify that the distance between two rational numbers on a number line is the absolute value of their difference.	

Instructional Focus Statements

Level 3:

Instruction should focus on students developing an understanding of and computing with rational numbers, including integers, positive and negative fractions, and positive and negative decimals. Students should encounter situations promoting positive and negative numbers combining to make zero in real-world situations and model this situation on a number line. Foundational to this is a student's ability to see real-world examples in terms of positive and negative numbers such as 30 degrees below zero is represented as -30˚. Modeling signed numbers on a number line enhances the idea of how a certain number of moves in a positive direction from zero combined with the same number of moves in the opposite direction ends at zero. Students should also understand that these are additive inverses because their sum is zero.

As students become more confident in working with additive inverses to make zero, they should use this technique to develop an understanding of how to add and subtract rational numbers using number lines, counters, and other visual representations. It is important that students are afforded the opportunity to develop this conceptual understanding as opposed to being presented with a set of "rules" for adding and subtracting signed numbers. Specifically with subtraction of rational numbers, one approach should be thinking of subtraction in terms of addition using the additive inverse. This results in defining the distance between two rational numbers on a number line as the absolute value of the difference between the two numbers. Students should then progress to discover and apply formal rules for adding and subtracting rational numbers with equations and support their reasoning with verbal and written explanations accompanied by visual models. In previous grade-levels students used the commutative, associate, and additive identity properties with whole numbers. Students should also be exposed to applying these properties when working with rational numbers. As students solidify their understanding of applying operations as strategies to add and subtract rational numbers, they should be able to solve real-world problems providing verbal and written justifications using mathematical language.

Level 4:

Students should be able to find the sum or difference of rational numbers in more complex equations. Additionally, when working in real-world situations, problems should be posed eliciting the unknown in any position and where multiple properties are embedded. To further extend understanding, students should reason about contextual problems that involve applying properties of operations to add and subtract rational numbers and interpret the respective sum or difference. This should be explained using precise mathematical language and supported by visual representations.

Standard 7.NS.A.2 (Major Work of the Grade)
Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
7.NS.A.2a Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
7.NS.A.2b Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.
7.NS.A.2c Apply properties of operations as strategies to multiply and divide rational numbers.
7.NS.A.2d Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Convert a fraction with a denominator that is a multiple of 10 to a decimal.	Convert fractions with a denominator of 2, 3, 4, 5, 6, 8, or 10 to a decimal. Choose a real-world context to represent a given product or quotient of rational numbers.	Multiply and divide a wide variety of rational numbers. Interpret products and quotients of rational numbers by describing real-world contexts. Apply properties of operations as strategies to multiply and divide rational numbers. Create a real-world context to represent a given product or quotient of rational numbers. Convert a rational number to a decimal using long division; know that the decimal form of rational	Create problems that involve multiplying and dividing rational numbers in real-world and mathematical problems and interpret the respective product and quotient. Make generalizations about long division quotients to determine if the decimal form of the rational number will repeat or terminate. Explain why multiplying or dividing a negative number by a negative number results in a positive answer using appropriate mathematical vocabulary.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		numbers terminates or eventually repeats.	Explain why multiplying or dividing a negative number by a positive number or vice versa results in a negative answer using appropriate mathematical vocabulary.

Instructional Focus Statements

Level 3:

Previously students have used visual models to represent addition and subtraction of rational numbers. When students begin to develop an understanding of multiplication and division of rational numbers, students must rely increasingly on their understanding of both the properties of operations and their knowledge of addition and subtraction with rational numbers. In order to build the necessary bridges from their previous understandings to situations where one or more of the numbers might be negative, they may rely on repeated addition or repeated subtraction in order to discover the relationships that exist when multiplying and dividing with signed numbers. Additionally, using real-world examples strengthens the understanding of multiplication and division of rational numbers. For example, students can understand that if 3 people owe a debt of $5, then the total debt owed is $15 resulting in the equation 3 x -5 = -15. Connecting multiplication to repeated addition can also support students in understanding the concept of multiplication with signed rational. In the same example, 3 x -5 = 15, can be represented as repeated addition as (-5) + (-5) + (-5) = -15. Students should employ their understanding that a negative number in terms of the opposite of a number, as learned in grade 6, and the use of the distributive property to deduct that (-1)(-1) can be represented as – ((1)(-1)) = -1.

When students are presented with division of signed rational numbers, they should make the connection that multiplication and division are inverse operations. In the example, 3 x -5 = -15, students should also know that the same is true for the division equation of -15 ÷ 3 = -5 or $\frac{-15}{3} = 5$. Students should interchangeably use notation for division as p ÷ q and $\frac{p}{q}$ and understand that division by zero is not defined. Students should use these connections to discover the rules for multiplying and dividing rational numbers. To extend students understanding of rational number division, students should use long division to convert rational numbers in fraction from to decimal form. Additionally, students should be able to sort the decimal form of rational numbers into two types, ending in 0 or terminating or repeating.

Level 4:

As students solidify their conceptual understanding of multiplication and division of rational numbers, they should be able to explain the relationship between the inverse operations. Students should also be able to explain in verbal and written form the connections between multiplication and division in real-world contexts explain how they discover a set of rules for signed numbers. Additionally, students should extend their understanding of converting a fraction to decimal form by making generalizations about long division quotients to determine if the decimal form of the rational number will repeat or terminate. This should be supported with verbal and written justification using precise mathematical language.

Standard 7.NS.A.3 (Major Work of the Grade)

Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Solve one-step mathematical problems involving addition and subtraction of positive rational numbers. Solve one-step real-world problems involving addition and subtraction of positive rational numbers. Apply the order of operations to evaluate numerical expressions with positive rational numbers. Choose a division problem that represents a complex fraction.	Solve two-step mathematical problems involving multiplication and division of positive rational numbers. Solve two-step real-world problems involving multiplication and division of positive rational numbers. Rewrite a complex fraction as a division problem. Apply the order of operations to evaluate numerical expressions with rational numbers and exponents.	Solve multi-step real-world problems involving the four operations with rational numbers. Solve mathematical problems involving operations with complex fractions. Solve real-world problems involving operations with complex fractions. Apply the order of operations (including grouping symbols and exponents) to solve mathematical and real-world situations involving rational numbers.	Create multi-step real-world problems that involve a wide variety of operations with rational numbers. Justify the solution path to solve real-world problems involving multiple operations with rational numbers. Explain the solution to a real-world problem in terms of its context.

Instructional Focus Statements

Level 3:

In grade 6, students used the order of operations to evaluate numerical expressions involving positive rational numbers and grouping symbols. In standards 7.NS.A.1 and 7.NS.A.2, students are developing a conceptual understanding of operations with rational numbers, including integers, positive and negative fractions, and positive and negative decimals. In standard 7.NS.A.3 students should extend this understanding to apply the four operations with rational numbers to solve real-world and mathematical problems.

Students should have multiple opportunities to interpret and write expressions and equations to represent the context of a wide variety of real-world problems. Problems should require multiple operations as well as positive and negative rational numbers. A variety of problems that require grouping symbols and continue to use the use of the distributive, commutative, associative, and additive identity properties should be included throughout instruction to help students learn to justify their manipulation of numbers. Students should be asked to explain why they chose the operations used in expressions and continue to use the order of operations to accurately evaluate those expressions. Classroom discourse should focus on what is happening in the context, rather than associating key words with certain operations which can sometimes be misleading. Students should be able to interpret the real-world problem in order to determine which operation(s) are necessary to solve the problem, including using models and other representations.

Problems that require division of rational numbers should include complex fractions $\frac{a}{b}$, where a and b can be fractions. Students should interpret a complex faction as the division of two fractions. Previously, students made the connection that multiplication and division are inverse operations when manipulating integers. As students extend this understanding they should have opportunities to discover that the relationship remains true with complex fractions. For example, $\frac{1/2}{1/4} = 2$ and $2 \; x \; ¼ = \; ½$. Solidifying the understanding of operations with a wide variety of rational numbers will be essential as students explore algebraic concepts in grade 8.

Level 4:

At this level, students should demonstrate a strong conceptual understanding of performing operations with rational numbers. Students should be given opportunities to create and solve multi-step, real-world problems that involve positive and negative rational numbers. Classroom discourse should focus on the meaning of the solution in terms of the context and the reasonableness of their solutions. Students should support their solution paths in multiple ways and fully explain all steps in the problem-solving process. A student who chooses to use a tape diagram to illustrate the problem-solving process might employ both a part-whole model and a comparative model to justify multiple operations required to arrive at an answer. Additionally, students should be challenged to consider alternative methods that are mathematically sound, which yield the same solution. In addition to finding solutions to these problems, students should be posed with questions that solicit an explanation of their solutions in terms of the context. These explanations should be presented using precise mathematical language.

Expressions and Equations (EE)

Standard 7.EE.A.1 (Major Work of the Grade)
Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Add and subtract linear expressions with positive rational coefficients. Choose equivalent expressions resulting from factoring or expanding linear expressions with positive whole number coefficients.	Choose equivalent expressions resulting from adding, subtracting, factoring, or expanding linear expressions with rational coefficients.	Add, subtract, factor, or expand linear expressions with rational coefficients.	Generate equivalent expressions resulting from adding, subtracting, factoring, or expanding linear expressions with rational coefficients and explain the reasoning using precise mathematical vocabulary.

Instructional Focus Statements

Level 3:

This standard is a continuation of coursework previously completed in 6.EE.A.3. In grade 6, students apply the properties of operations to generate equivalent expressions. For example, students apply the distributive property to the expression 3(2 + x) resulting in 6 + 3x and y + y + y in resulting in 3y to produce equivalent expressions. In grade 7, students are introduced to operations with positive and negative integers and should employ this understanding and their prior knowledge of properties of operations (distributive, commutative, associative, identity, and inverse properties) of addition and multiplication as strategies for adding, subtracting, factoring, and expanding linear expressions. When applying properties, it is imperative that students have a strong conceptual understanding of using the order of operations when producing equivalent linear expressions. In this standard, coefficients can include rational numbers that encompass integers, positive/negative fractions, and decimals. This standard reinforces foundational skills for grade-level coursework in which students rewrite expressions to reveal specific quantities and will lead to future coursework in which students will work with other expression types other than linear expressions.

Level 4:

Students should develop an in-depth understanding of applying properties of operations and be flexible in selecting the most efficient operations to use and explain their reasoning using precise mathematical vocabulary. Students should also be able to justify their solutions by substituting values into equivalent expressions to verify that both expressions result in the same value and are therefore equivalent expressions, and their solution path is correct. Additionally, students should extend their knowledge of applying properties for operations as strategies to add, subtract, factor, and expand liner expressions as a part of solving contextual problems. This will be a foundational skill for future coursework in which students will work with other expression types.

Standard 7.EE.A.2 (Major Work of the Grade)
Understand that rewriting an expression in different forms in a contextual problem can provide multiple ways of interpreting the problem and how the quantities in it are related.

For example, shoes are on sale at a 25% discount. How is the discounted price P related to the original cost C of the shoes? C - .25C = P. In other words, P is 75% of the original cost for C - .25C can be written as .75C.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose an expression that represents a given real-world situation. Identify independent and dependent variables in a contextual problem.	Choose a form of an expression that accurately highlights a particular relationship between quantities in a problem.	Generate an equivalent expression to highlight a given relationship within the problem, given a contextual problem and an expression that represents it. Choose multiple forms of an expression and explain how each accurately highlights a particular relationship between quantities in a problem.	Generate an expression that accurately highlights a particular relationship between quantities in a contextual problem. Explain the relationship that exist between different expressions representing the same contextual problem and write a justification as to when each might be more beneficial.

Instructional Focus Statements

Level 3:

Students are developing a foundational understanding that rewriting an equivalent expression can better show the relationship among the terms in an expression. Students should be presented with real-world problems that can be modeled with more than one expression. Additionally, students should engage in discussions about why one expression is better than another equivalent expression when both are generated from the same contextual problem. By initially using simple contexts that have more accessible entry points, students will be allowed the opportunity to conceptually develop an understanding of the connections that exist between the context and various forms of equivalent expressions. Additionally, by the end of grade 7, students should be able to articulate and defend the choices that they make for expressions representing contextual problems.

Level 4:

Students should be able to generate an expression resulting from a real-world problem and explain the parts of the expressions in terms of the context. Students should also be able to generate multiple equivalent expressions and justify the use of one over the other. Additionally, they should be able to provide an explanation of the connection that exists between the resultant expressions. Justifications and explanations should be in both verbal and written form using precise mathematical vocabulary.

Standard 7.EE.B.3 (Major Work of the Grade)
Solve multi-step real-world and mathematical problems posed with positive and negative rational numbers presented in any form (whole numbers, fractions, and decimals).
7.EE.B.3a Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate.
7.EE.B.3b Assess the reasonableness of answers using mental computation and estimation strategies.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Solve single-step mathematical problems with positive rational numbers presented in a single form. Conversion between forms of rational numbers is not required to solve the problem.	Solve single-step mathematical problems involving positive and negative rational numbers written in a single form. Conversion between forms of rational numbers is not required to solve the problem.	Solve multi-step, real-world and mathematical problems with rational numbers where conversion between forms of rational numbers may or may not be required.	Solve multi-step, real-world and mathematical problems with rational numbers where conversion between forms of rational numbers is required. Assess the reasonableness of answers resulting from multi-step, real-world and mathematical problems with rational numbers presented in any multiple forms.

Instructional Focus Statements

Level 3:

Students should solve multi-step real-world and mathematical problems that contain integers, fractions, and decimals. Students should also use previously acquired skills around converting fractions, decimals, and percentages and use properties of operations to find equivalent forms of expressions when needed. Students should solidify their understanding by checking their solutions for reasonableness using estimation strategies such as rounding, compatible numbers, and benchmark numbers.

Level 4:

Students should extend their understanding of solving multi-step real-world and mathematical problems by assessing solutions for reasonableness when working with rational numbers presented in any form. Additionally, they should be able to provide an explanation of their solution path, reasoning for their estimation strategy, and how their solution connects to the contextual problem. Justifications and explanations should be in both verbal and written form using precise mathematical vocabulary.

Standard 7.EE.B.4 (Major Work of the Grade)

Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

7.EE.B.4a Solve contextual problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

7.EE.B.4b Solve contextual problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality on a number line and interpret it in the context of the problem. *(Note that inequalities using >, <, ≥, and ≤ are included in this standard.) For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	**Students with a level 2 understanding of this standard will most likely be able to:**	**Students with a level 3 understanding of this standard will most likely be able to:**	**Students with a level 4 understanding of this standard will most likely be able to:**
Identify the variable quantity in a real-world or mathematical situation. Choose an equation in the form p + q = r or px =r to represent a given contextual problem. Choose the solution for equations of the form px + q = r and p(x + q) = r when p, q, and x are all whole numbers. Choose a number line graph that represents the solution set for an inequality in the form x + q > r or x + q < r.	Choose an equation in the form px + q = r or p(x + q) =r to represent a given contextual problem. Choose a number line graph that represents the solution set for an inequality in the form px + q > r or px + q < r. Choose an inequality in the form px + q > r or px + q < r that represents a given contextual problem.	Solve contextual problems leading to equations in the form px + q = r or p(x + q) =r. Solve contextual problems leading to inequalities in the form px + q > r or px + q < r. Graph the solution set for an inequality in the form px + q > r or px + q < r on a number line. Interpret the solution set for an inequality in the form px + q > r or px + q < r in the context of the problem.	Solve contextual problems leading to equations in the form px + q = r or p(x + q) =r both algebraically and arithmetically and identify the similarities and differences between both approaches. Solve contextual problems leading to inequalities in the form px + q > r or px + q < r, where p, q, and r are specific rational numbers and identify and justify the operations used in each approach. Interpret the solution set for an inequality and explain the reasonableness of the solution set with respect to the context.

Instructional Focus Statements

Level 3:

Students should develop fluency solving contextual problems that can be modeled by linear equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. As students extend their understanding, they should work with problems that can be solved arithmetically but can also generalize an algebraic solution that can be applied to the contextual situation.

Students should use the foundational process of solving equations to solving word problems with inequalities. Students should understand that similar properties are used in solving both equations and inequalities. Students should also recognize one important new consideration in solving inequalities: multiplying or dividing both sides of an inequality by a negative number reverses the order of the comparison it represents. It is useful to present contexts that allow students to make sense of this. Students should also be able to graph their solution set on a number line and interpret the meaning of their solution set with respect to the context of the problem.

Level 4:

Students should extend their understanding of solving contextual problems to recognize whether the problem should be represented with an equation or an inequality and provide justification for their reasoning. Also, students should be able to interpret the solution or solution set and determine its reasonableness to the contextual situation. As students solve equations and inequalities, they should be able to justify their solution or solution set by substituting values in the equation or inequality to determine if the equation or inequality is true. Justification for this should be in written and verbal form using precise mathematical vocabulary.

Geometry (G)

Standard 7.G.A.1 (Supporting Content)
Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose the actual lengths of the geometric figure, given a scaled geometric figure and the scale factor that generated it.	Identify the length of the missing side, given two geometric figures with an unknown side measure and the scale factor that relates the two figures.	Solve problems involving scale drawings of geometric figures. Reproduce a scale drawing at a different scale.	Solve a wide range of problems involving scale drawings of geometric figures. Explain the process of generating scaled drawings using precise mathematical vocabulary.

Instructional Focus Statements

Level 3:

Students should be able to work with scale drawings by reading them, calculating the scale, computing actual lengths from a scale drawing, and reproducing a scale drawing using another scale. Students should apply the concept of proportionality when developing a conceptual understanding of scale drawings. Students should be provided with problems that require them to calculate actual measures after they have generated a scale drawing such as area, perimeter, and volume from scale drawings using both whole number and fractional measurements. It is essential that students understand that when you multiply the dimensions of one scale drawing by a scale factor, you produce another unique scale drawing.

Level 4:

Students should employ their understanding of proportionality in order to demonstrate their conceptual understanding of the connections that exist between measurements in real-world problems and those in their scale drawings. Students should also be able to redraw scale drawings using different scales and explain their reasoning using precise mathematical language when presenting solutions to real-world problems. Students will extend this concept in future grades when working with proportional relationships with similar triangles and figures.

Standard 7.G.A.2 (Supporting Content)

Draw geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Recognize examples of geometric shapes with given conditions.	Choose a geometric shape that represents a given set of conditions.	Construct geometric shapes with given conditions. Recognize when a set of segments cannot form a triangle. Recognize when conditions form a unique triangle, more than one triangle, or no triangle.	Explain why a set of conditions determines no triangle, a unique triangle, or more than one triangle. Create a set of conditions that form a unique triangle, more than one triangle, or no triangle and explain the reasoning using precise mathematical language.

Instructional Focus Statements

Level 3:

In learning this standard, students should drawing geometric shapes with a variety of tools, including technology, rulers, protractors, and free-hand. The focus of this standard is on triangles and constructing them from three given sides or angles. Students should be able to determine when given three measures if these conditions result in one, more than one, or no triangle. Students should discover with visual representations or models that the sum of the angles in a triangle is 180˚. Students should also discover that the sum of two side lengths of a triangle is always greater than the third side. This understanding can be nicely integrated as students work with solving simple inequalities. Additionally by sketching geometric shapes that adhere to a given set of conditions, students lay the foundation for the concepts of congruence and similarity in grade 8, and for the practice of geometric deduction that will grow in importance throughout the high school course work.

Level 4:

As students deepen their understanding, they should be able to make generalizations about constructing triangles given a set of conditions. In doing so, students should be able to not only identify that a given set of conditions is one, more than one, or no triangle, but also create examples to support their generalizations.

Standard 7.G.B.3 (Supporting Content)

Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the parts of a circle that represent the radius, diameter, area and circumference. Identify the circumference as a perimeter measurement of the distance around a circle. Find the area of a circle when given the correlating version of the formula and the radius or diameter. Find the circumference of a circle when given the correlating version of the formula and the radius or diameter. Find the diameter given the radius or radius given the diameter. State that pi has an approximate value of 3.14.	Identify the formula used to find area of a circle. Identify the formula used to find circumference of a circle. Determine the circumference of a circle given either the radius or diameter. Determine the area of a circle given either the radius or diameter. Informally explain the relationship between circumference and diameter.	Determine the radius and/or the diameter of a circle when given the circumference. Determine the radius and/or the diameter of a circle when given the area. Determine the area of a circle when given the circumference and vice-versa. Solve a real-world problem that involves determining the area of a circle. Solve a real-world problem that involves determining the circumference of a circle. Express the area and circumference of a circle in terms of pi and as an approximate numerical value. Informally explain the relationship between the circumference and area of a circle.	Justify the relationship that exists between circumference and area written in mathematical explanations. Create and solve a real-world problem which involves calculating the area and/or circumference of a circle.

Instructional Focus Statements

Level 3:

Students have worked with the concepts of area and perimeter since grade 3. Prior to grade 7, this experience was limited to polygons. Students now build on that understanding to solve problems involving the area and circumference of circles. Because students are often confused by the formulas for the area and circumference of a circle, instruction should focus on helping students discover the relationship between the two. Prior to providing the formula, students should be given ample opportunity to discover the mathematical relationship that exists between the circumference of a circle and its diameter as well as the mathematical relationship that exist between the area of a circle and its radius. Activities that allow students to repeatedly measure and compare the circumference and diameter of different-sized circles will lead them to discover that circumference is approximately three times the size of the diameter. With this discovery the teacher should introduce the concept of pi and its application.

Students are expected to informally derive and explain the relationship between the circumference and area of a circle. One way to model this is by slicing a circle into thin pie-shaped pieces and arranging the pieces to form a shape similar to a parallelogram. Students should use their prior knowledge of finding the area of a rectangle or parallelogram, relating it to the parts of the circle (before it was transformed). From these opportunities, teachers should facilitate a discussion to lead students to the formulas for circumference and area of a circle. Students should discover that the height of the parallelogram is r, the radius of the circle. Students should also discover that the length of the parallelogram is equal to one-half of the circumference of the circle, therefore, the area of the parallelogram is $r \cdot \frac{1}{2} (2\pi r) = \pi r^2$. Additionally, it is important that students understand exact answers versus approximate answers when rounding the value of pi. Students should be expected to give solutions in terms of pi and as approximate decimal values.

It is important that students are exposed to activities that allow them to apply the formulas to solve a wide variety of mathematical and real-world problems. This means that they should sometimes be presented with problems that provide the diameter, rather than the radius. Discussion around area should include the circle as a whole, as well as quarter circles and semi-circles. Students should also be challenged to use variations of the area and circumference formulas to solve for any part of the circle when given the other part(s). For example, if students are given the circumference, they should also be able to find the radius or diameter.

Level 4:

Students at this level should be able to demonstrate conceptual understanding of the relationship between the circumference and area of a circle. By building on the development of their prior knowledge of area and perimeter, students should be able to derive the formulas for the area and circumference of a circle. Students should also be challenged to use and manipulate a model of a circle to explain, using precise mathematical vocabulary, the relationship between circumference and area of a circle and how that relationship is used to derive the formulas for each.

Standard 7.G.B.4 (Supporting Content)

Know and use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Classify angles using the terms supplementary, complementary, vertical, and adjacent. Define the relationship of supplementary, complementary, vertical, and adjacent angles.	Determine the measurement of an unknown angle based on the properties of supplementary angles. Determine the measurement of an unknown angle based on the properties of complementary angles. Determine the measurement of an unknown angle based on the properties of vertical angles.	Apply the properties of supplementary or complementary angles to write an equation to solve for unknown angles. Apply the relationship of vertical angles to write an equation to solve for unknown angles. Apply the properties of adjacent angles to write an equation to solve for unknown angles. Solve equations written using the relationships of angles to find the measurement of an unknown angle.	Create problems related to finding unknown angles in a real-world setting. Justify the solutions to problems involving angle relationships using precise mathematical language.

Instructional Focus Statements

Level 3:

In grade 5, students learned to draw angles and classified them as right, acute, obtuse, straight, or reflex. In grade 7, the focus shifts to the angles formed by intersecting lines, vertical angles, and adjacent angles and their relationships. Students should be provided opportunities to explore supplementary, complementary, vertical, and adjacent angles and their relationships to one another through measuring and finding patterns in order to discover their definitions. For example, when students are given two adjacent angles that are shown to form a straight line, they should be given the opportunity to use measuring tools to discover that the sum of the angles are equivalent to 180 degrees. Students should be expected to use these angle relationships to

write and solve multi-step problems by applying what they know about the different types of angles in both intersecting lines and polygons. Discussion should lead students to find multiple angle relationships from a context and justify their reasoning and solutions. By substituting or measuring with a protractor, students can initially verify solutions to these equations, or the measure of an unknown angle. This standard is supported by students' prior learning (standard 6.EE.B.4), where the emphasis is on writing an equation that represents the relationships of the angles. A strong understanding of angle relationships will be essential as students later use informal arguments to establish facts about angles in Grade 8 (standard 8.G.A.3).

Level 4:

As students strengthen their understanding, they should make connections from intersecting lines and polygons to real-world contexts With minimal information, students would be able to write and solve multiple equations that represent the angles formed to solve for the measurement of the missing angles within the context. Students should be able to justify their equations and solution paths using precise mathematical language.

Standard 7.G.B.5 (Supporting Content)
Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Find area of triangles in real-world and mathematical problems. Find area of quadrilaterals in real-world and mathematical problems.	Find the surface area of rectangular prisms, including cubes, in real-world and mathematical problems. Find the volume of rectangular prisms, including cubes, in real-world and mathematical problems. Find the area of complex two-dimensional polygons composed of rectangles and/or triangles. Find the surface area of a triangular prism in mathematical problems. Find the volume of a triangular prism in mathematical problems.	Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	Solve for missing dimensions when given the area or volume of an object. Use precise mathematical language to explain the relationships between area, volume, and surface area. Derive formulas for finding the surface area of a cube or right prism.

Instructional Focus Statements

Level 3:

In grade 6 students composed and decomposed figures to find the area of various shapes and were introduced to finding the surface area of nets made up of rectangles and triangles. Building on these concepts to construct the area of new figures and extend this to three dimensional figures will lead students to a conceptual understanding of volume and surface area rather than rote memorization of formulas. Students need to explore the concepts of area, surface area, and volume with concrete materials where they are encouraged to measure to find area, fill to find volume, and use nets to determine

surface area. Experiences with surface area should be focused on the building of shapes using nets as experienced in previous grades. By working with nets and finding the total of the area of the faces, students can be introduced to the formulas for surface area, but overall understanding should be conceptual in nature.

To build on the prior knowledge students have with volume through the use of unit cubes, instruction should focus on the idea that volume is the space an object holds, calculated by the area of the base multiplied by its height. Instruction should focus on this conceptual understanding to find the volume of right rectangular and right triangular prisms by identifying the bases of prisms and the use of the area. Once students have a strong understanding of the concept of volume for various base shapes, they can then translate that to the formula format of V=Bh, with emphasis on the B representing the area of the base shape.

Students should use formulas to solve a variety of problems where they must calculate the areas of rectangles and triangles that comprise the three-dimensional figures in both real-world and mathematical problems. Students should then be expected to apply this knowledge to solve problems in real-world contexts where solid shapes have various rectangular and triangular prism components.

Level 4:

Students at this level should be expected to find the area, volume, and surface area of complex polygons that are presented in a visual mathematical or real-world context. Students should also be challenged to apply their knowledge of area, volume, and surface area to find missing dimensions in two and three-dimensional shapes. Problems could be presented where the volume and the area of the base are given and students are challenged to find the missing height.

In order to further the conceptual understanding, instruction should include opportunities for discourse where students are given complex three-dimensional shapes that involve various combinations of rectangular and triangular prisms together. Students should be expected to justify the formulas they used to find volume and surface area and connect them to parts of the complex object. Likewise, they could generate context for a two- or three-dimensional polygon for others to analyze and solve. Discourse should be facilitated so that students connect the context to the object and provide explanations to support the use of each formula.

Students should be able to generate and use nets to derive the formulas for surface area of a cube or right prism, and apply these formulas to solve real-world problems. Students should be expected to use precise mathematical language to justify the relationships that exist between area, volume, and surface area.

Statistics and Probability (SP)

Standard 7.SP.A.1 (Supporting Content)
Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Define random sampling. Collect sample data from a population. Generate a statistical question in a real-world context.	Explain the importance of random sampling in statistics. Differentiate between a sample and population. Recognize invalid sampling techniques.	Determine if a sample is representative of a population. Determine if a sample is biased. Identify factors that would contribute to bias. Generate random samples that are representative of a larger population.	Conduct a survey and explain why the method used will produce a sample that is unbiased and representative of the population. Explain adjustments needed to produce a representative sample when given an invalid sampling technique, Critique examples of random sampling as statistical tools using precise mathematical vocabulary.

Instructional Focus Statements

Level 3:

In grade 6, students formulated statistical questions and collected data to answer those questions. In grade 7, students improve the production of data and understand the importance of selecting random samples. This standard introduces students to random sampling and how those samples can be used to gather information about the populations from which they are drawn. Discourse should be facilitated, leading students to an understanding of statistics, why it is studied, and how it is useful in real-world applications. Opportunities to collect relevant data from a large population should reveal the difficulty in gathering statistics on an entire population. To emphasize this point, students learn that sampling is the process of taking a subset of subjects

that is representative of the population and collect data on that subset. Students should be exposed to statistical questions that would require random sampling and could not be easily answered using other sources, such as the Internet. Providing opportunities for students to analyze statistical data while considering bias and whether the sample is representative of the population is essential. In order to show the benefits of randomization, students should be presented with scenarios that include random and non-random samples and discuss how each might affect the validity of the results drawn from the samples. While many students will understand the need to produce a random sample, exposure to situations that are random but not representative of the population, will provide opportunities for discourse using precise mathematical vocabulary around the use of both conditions. In addition, instruction should include discussions around sample sizes so that students understand how samples that are too small can skew results.

Level 4:

At this level, students should be challenged to go beyond generating random samples, to explaining the statistical process used to generate their samples, using precise mathematical vocabulary. Instruction should include opportunities for students to justify their own sampling and explain why the sample is representative of the population. When given an invalid sampling technique, students at this level should be able to explain why it will not produce a representative sample and explain any adjustments that need to be made to the process such as increasing the size of the sample or eliminating opportunities for bias.

Standard 7.SP.A.2 (Supporting Content)

Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Explain the benefits of random sampling. Differentiate between a sample and population. Determine if a sample is representative of a population.	Generate multiple samples that are representative of a larger population. Describe a sample using a measure of center or variation. Differentiate between sample measures and population measures.	Recognize that increasing sample size affects sampling variability. Make inferences and generalizations about a population using data from random samples. Justify inferences and generalizations using appropriate mathematical vocabulary. Recognize how sample variations affect the accuracy of predictions.	Explain the impact of sample size on sampling variability with precise mathematical language. Assess the accuracy of a prediction using multiple random samples.

Instructional Focus Statements

Level 3:

In grade 6, students formulated and answered statistical questions. In grade 7, students are introduced to random sampling and learn how those samples can be used to gather information about the populations from which they are drawn (standard 7.SP.A.1). Instruction should include multiple opportunities to collect data samples and make generalizations about populations. This can be done using actual experimentation or simulation methods. Discourse should focus on the determination of the sample mean or proportion, and students should be encouraged to think about how different samples would result in different values. This should be explained as sampling variability and instruction should include opportunities for students to understand and defend why it must be considered when using sample data to learn about a population.

Students should engage in experiments and simulations that illustrate how different samples from the same population can differ to solidify understanding of sampling variability. For example, a sample of 25 survey responses might yield a different proportion than a second sample of 25 survey responses. In addition to noticing variations in samples, students should be expected to explain variations in samples. As students collect and use multiple samples of data to make generalizations about a population, issues of variation in the samples will provide opportunities for discussion and may lead to further investigations.

Activities that allow students to compare sample measures to population measures are essential for students to recognize and understand that the two are not the same. Engagement opportunities should reveal that conclusions drawn from random samples generalize beyond the sample to the population from which the sample was drawn; however, these are only estimates and there will be some discrepancy between the two. Combining sample data on a dot plot will provide a visual illustration depicting how the sample measures vary. Questions should be posed so that students notice variation in estimates and can estimate about how far off the prediction about the population could be. This experience in collecting and analyzing univariate data will be beneficial as students go on the collect and represent bivariate data in grade 8.

Level 4:

Students at this level should be challenged and expected to move beyond recognizing sample variability on a dot plot by explaining variability between sample measures using precise mathematical language. Students should justify placement of dots and lead others to see how close the sample measure is to the population measure. When presented with sample measures, students should be challenged to explain how sample size could affect variability. When posed with objections, students should be encouraged to use multiple sample sizes to show how a change in sample size results in a change in variability. When given a prediction about the winner of a school election, students at this level should be expected to assess the prediction based on sample data. Students should be challenged to use the variation in sample data to explain how the prediction is, or is not, justified.

Standard 7.SP.B.3 (Supporting Content)

Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team; on a dot plot or box plot, the separation between the two distributions of heights is noticeable.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Recognize when an overlap exists between two data sets. Determine the center of numerical data sets.	Represent real-world data sets using dot plots and box plots. Solve problems involving data from a dot plot or box plot.	Compare the graphs of two data sets with similar variability to determine the degree of overlap. Recognize that an increase in variability can increase the overlap in data. Model and compare two real-world data sets by measuring the difference between their centers and expressing it as a multiple of a measure of variability.	Determine and explain when the difference in two data sets is meaningful. Use graphs to explain how an increase in variability can increase the overlap in data.

Instructional Focus Statements

Level 3:

In grade 6, students displayed single data sets in various ways and described data using measures of center and spread. In grade 7, students build on this learning to compare data sets of two distinct populations. This is the students' first experience with comparing two data sets, so instruction should begin with the analysis of one data set and then add a second for comparison. Discussion should be facilitated so that students consider measures of variability as well as mean or median. Instruction should also include opportunities to engage in discussion about the variability of both data sets. Student should be exposed to examples with both increased and decreased variability, leading them to conclude that an increase in variability can increase the overlap in data. Using a given context, instruction should include discourse around the overlap in data and what it means in terms of the context. Students should recognize that when comparing the means of two data sets, the mean of each sample will most likely not be the same.

It is important that students are exposed to contextual examples, which will allow them to not only calculate measures, but also understand their meaning in terms of the context. When students initially begin to describe the degree of overlap in data, displaying the two graphs vertically and aligning the scales could make the comparison more visible. Students will have prior knowledge of graphs, numerical data that is of interest to them (e.g., sports, government, recent events) should be incorporated and provided when possible. When comparing large data sets, calculators can be used to aid in computations so that student discussion is focused on the comparison of the data sets. These understandings will be essential as students go on to model bivariate relationships in grade 8.

Level 4:

In addition to comparing data sets, students at this level should be challenged to determine and explain if the difference in the two data sets is meaningful or great enough to assert a distinction between the data sets. Given two raw data sets, students should be challenged to draw conclusions based on the difference of the means. For example, if there is only a small difference in the mean prices of the two stores, one might conclude that there is no real benefit in shopping at one or the other. Students should be challenged to justify this conclusion using precise mathematical language and appropriate graphs. Students should also be challenged to explain how an increase in variability can increase the overlap in data. Instruction should include opportunities for students to generate multiple samples and illustrate the change in variability using appropriate graphs.

Standard 7.SP.B.4 (Supporting Content)

Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a 7th grade science book are generally longer than the words in a chapter of a 4th grade science book.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Describe a sample using measures of center. Describe a sample using measures of variability.	Determine the appropriate measure of center or variability for describing a single data set. Make inferences about a single population using data from random samples. Draw comparative conclusions about two populations using graphs.	Determine the appropriate measure of center or variability for comparing two data sets. Draw valid comparative inferences about two populations using measures of center. Draw valid comparative inferences about two populations using measures of variability. Identify valid inferences related to the comparison of two data sets.	Critique the selection of a measure of center or variability that is used to compare two data sets. Critique comparative inferences about two populations using appropriate mathematical vocabulary.

Instructional Focus Statements

Level 3:

In grade 6, students represented data using graphs and determined measures of center and variability for single data sets. Building on standard 7.SP.A.2, students now compare two sets of data using appropriate measures of center and variability. Instruction should include opportunities to analyze data sets that are presented in multiple formats (e.g., dot plots, box and whisker plots, etc.). In comparing the data sets, students should have opportunities to determine which measures of center or variability should be used for a valid comparison. Students should make comparative inferences about the data sets using precise vocabulary. Rather than just determining measures of center and variability, it is important that students use them to support their

comparative inferences and engage in discussion with their peers about around the validity of their inferences. An understanding of statistics and comparative inferences is necessary as students go on to compare center and spread of two or more different data sets in high school.

Level 4:

Students at this level should have a thorough understanding of measures of center and variability and how they are used to compare data sets. Students should be challenged to critique the selection of a measure of center or variability that is used by a peer to compare two data sets. In addition, students should be challenged to make adjustments needed to create a more appropriate comparison. Multiple opportunities to critique comparative inferences will be beneficial in building students' ability to make decisions using multiple data sets.

Standard 7.SP.C.5 (Supporting Content)

Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Define probability as the likelihood of a chance event occurring.	Define the complement of an event as the chance of the event not occurring. Recognize that the probability of a chance event can be represented as a rational number from 0 to 1.	Understand that an impossible event has a probability of 0 and that a certain event has a probability of 1. Understand that a probability closer to 0 represents an unlikely chance event, a probability close to 1/2 represents an equally likely (to occur or not occur) chance event and a probability closer to 1 represents a likely chance event. Understand that a chance event and its complement has a sum of 1. Conduct simple experiments and calculate probabilities as fractions, decimals, and/or percents. Determine where probabilities of simple events lie on the probability scale.	Justify the categorization of events as likely, unlikely or equally likely (to occur or not occur) using precise mathematical vocabulary. Use reasoning to determine and explain where a probability lies on the scale when expressed as a fraction such as 3/8.

Instructional Focus Statements

Level 3:

In grade 6, students used ratio and rate reasoning to solve real-world and mathematical problems. In grade 7, they expand their understanding of ratios and develop and understanding of probability. This is the students' first formal introduction to probability. Students should recognize that the probability of any single event can be expressed in terms such as impossible, unlikely, equally likely, likely, or certain. Discussion should include familiar events in students' lives and the likelihood of those events occurring. Initial discussions should relate the likelihood of these events to probabilities of 0, ½ and 1. Students should engage in discourse around the meaning of numerical probabilities, understanding that the closer the probability is to 1, the greater the probability the event will occur.

Since students were exposed to the number line in previous grades, they should have multiple opportunities to place probabilities on the number line, or probability scale. Instruction should progress to include rational number probabilities between 0 and 1 so that students understand that a probability can be expressed as a fraction, decimal or percent. It is important to discuss why probability cannot be expressed as a number greater than 1. Conducting simple experiments (e.g., flipping a coin, tossing dice, spinning a spinner, etc.) will solidify students' understanding of probability and allow them to connect numerical probabilities to verbal descriptions. This initial understanding of probability will be essential as students go on to experiment with relative frequencies.

Level 4:

Students at this level can categorize events, but can also justify the categorization of those events using precise mathematical vocabulary and the probability concept. Students should be challenged to determine and explain where a probability lies on the scale when it is presented as a non-benchmark fraction. Students at this level can justify their placement of these probabilities by reasoning about their proximity to benchmark fractions.

Standard 7.SP.C.6 (Supporting Content)

Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Recognize that the probability of a chance event can be represented as a rational number from 0 to 1.	Conduct simple experiments and calculate probabilities as fractions, decimals and/or percents. Recognize relative frequency as the observed number of successful events for a finite sample of trials.	Collect data on chance events by conducting experiments and approximate the relative frequency of an event given the probability. Differentiate between relative frequency and theoretical probability. Determine theoretical probability and relative frequency in real-world situations.	Design and conduct probability experiments, making conjectures about the outcomes using precise mathematical vocabulary. Justify conjectures made about experimental probabilities.

Instructional Focus Statements

Level 3:

In grade 7, students are formally exposed to probability for the first time. While students developed an understanding of the meaning of probability with standard 7.SP.C.5, they now learn the difference between theoretical probability and relative frequency (experimental probability). Instruction should include multiple opportunities to collect data on chance events and make conjectures about the probability of those events. Instruction should also include opportunities to pool data from experiments and observe long-run frequencies in small groups. Discussion should focus on the idea that as the number of trials increase, the relative frequency (experimental probability) approaches the theoretical probability. The use of simulation with technology will be beneficial as students learn to make predictions about the relative frequency of events.

Level 4:

At this level, students have a strong understanding of relative frequency and have had multiple opportunities to conduct experiments to observe long-run relative frequency. In addition to participating in experiments, students should be challenged to design simulations, then summarize the data as experimental probabilities and make conjectures about theoretical probabilities. Additionally, students should have multiple opportunities to compare their predictions to the experimental outcomes, as well as justify and refine their conjectures about theoretical probability using precise mathematical vocabulary.

Standard 7.SP.C.7 (Supporting Content)

Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

7.SP.C.7.a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.

7.SP.C.7.b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Conduct simple experiments and express probabilities as fractions, decimals and/or percents. Recognize relative frequency as the observed number of successful events for a finite sample of trials.	Approximate theoretical probability and relative frequency in real-world situations. Recognize that probability estimates are affected by sample size. Understand the difference between uniform and non-uniform probability models.	Develop appropriate probability models to find probabilities of events with equally likely outcomes. Develop appropriate probability models to find probabilities of events with outcomes that are not equally likely. Compare probability models to observed frequencies and explain any discrepancies between the model and observed frequencies. Recognize when discrepancies between theoretical probability and relative frequency indicate an error.	Explain orally and in writing any discrepancies between probability models and observed frequencies using precise mathematical vocabulary.

Instructional Focus Statements

Level 3:

In grade 7, students learn that probabilities are useful for predicting what will happen over the long run. Applying what they learned about theoretical probability, students predict frequencies of outcomes using appropriate probability models. The probabilities of the models can be either theoretical or experimental with outcomes that may or may not be equally likely. For example, the probability of a balanced coin landing on heads or tails is ½, but the probability of a tossed thumbtack landing point up is not necessarily ½ as the two possible outcomes may not be equally likely. Students should also develop models for geometric probability, utilizing their understanding of geometric concepts. For example, given a circle inscribed in a square, determine the probability of choosing a point that is in the circle.

Instruction should include multiple opportunities to conduct experiments so that students recognize appropriate models and understand that probability estimates are affected by sample size. Students should also be provided multiple opportunities to replicate experiments, comparing the results to theoretical probabilities. Discussion should lead students to realize that discrepancies between theoretical probability and relative frequency do not necessarily indicate an error/inaccuracy. Experiments can be conducted using physical objects but can also be conducted using random generation devices such as bag pulls, spinners, number cubes, coin toss, and colored chips. Extensive practice with experiments will be beneficial as students go on to represent sample spaces for compound events in grade 8.

Level 4:

In addition to recognizing discrepancies between probability models and observed frequencies, students at this level should be able to explain any discrepancies orally and in writing using precise mathematical vocabulary. Students should have multiple opportunities to perform experiments, comparing the relative frequency to the known theoretical probabilities. In comparing the two, students should engage in discourse around the significance of the discrepancy. Students should be able to explain that small differences in the relative frequency and theoretical probability does not indicate an error. For example, when conducting 20 trials a relative frequency of 9/20 is close to 1/2 and does not indicate an error. In the event there is a true discrepancy, students should be expected to provide reasons for the discrepancy. One possible reason could be that there were not enough trials conducted in the experiment. A student at this level should be able to explain why more trials are needed and conduct those trials to show how the discrepancy could be resolved.

Standard 7.SP.D.8 (Supporting Content)
Summarize numerical data sets in relation to their context.
7.SP.D.8.a. Give quantitative measures of center (median and/or mean) and variability (range and/or interquartile range), as well as describe any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
7.SP.D.8.b. Know and relate the choice of measures of center (median and/or mean) and variability (range and/or interquartile range) to the shape of the data distribution and the context in which the data were gathered.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Summarize data sets using measures of center. Create dot plots, histograms, and box plots from numerical data.	Summarize data sets using measures of variability. Interpret dot plots, histograms, and box plots in relation to the context of the statistical question.	Determine the most appropriate measure of center and variability to summarize data derived from real world context. Use graphs to describe patterns in data derived from real world context. Recognize how measures of center and variability affect the shapes of data distributions.	Construct viable arguments to explain the selection of statistical measures, predicting how changes in the data affect those statistical measures.

Instructional Focus Statements

Level 3:

In grade 6, students summarize numerical data sets using measures of center and variability while describing patterns with reference to the context in which the data were gathered. Students in grade 6 also related the choice of measures to the shape of the data distribution and the context in which the data were gathered. In grade 7, students build on their understanding of these measures to not only summarize data sets, but also describe any significant deviations from the overall pattern and expand their choice of measure to include variability. Instruction should include opportunities to summarize numerical data derived from statistical questioning using quantitative measures of center and variability mean, median, range, quartiles, and interquartile ranges. Students should be led to realize that the measure of center chosen to describe a data set will depend upon the shape of the data distribution and

context of data collection. Students gained an understanding of the mean in grade 6 but should understand that it can be affected greatly by very low or very high data points. Exposure to data sets with outliers should lead students to recognize when a difference measure might be more descriptive and appropriate for the given data set. Additionally, exposure to data sets that are not symmetrically distributed should lead students to recognize when data sets are skewed and better represented with a measure other than the mean. To show variation of data, students should engage in activities involving the interpretation of data represented by graphs such as line plots, histograms, and box and whisker plots. Multiple opportunities to summarize data sets in relation to the context will be essential in solidifying students' ability to select an appropriate measure. Discourse should be facilitated so that students have opportunities to justify the measure of center or variability chosen. A thorough understanding of data distributions and patterns will be beneficial as students go on to investigate bivariate measurement data in grade 8.

Level 4:

Students at this level should have a strong understanding of measures of center and variability and be able to determine why a measure was chosen to summarize numerical data sets. Students also understand that the way data is collected, organized, and displayed influences interpretation and can inform decisions. Instruction should include opportunities to explain how a change in the data set would affect the chosen measure. For example, the removal of an outlier could prompt students at this level to change the chosen measure of center and justify the change in the chosen measure. Students should also be expected to use line plots, histograms, and other data displays to explain how data can be misleading both intentionally and unintentionally.