The following example problem is for the design of a rip rap lined channel. This design is based upon U.S. Department of Transportation – Federal Highway Administration: Hydraulic Engineering Circular Number 15 (HEC15)and involves an iterative p Note that many designs typically have design constraints such as limited easement width or right of way. Each design must be consistent with the site layout and must clearly address the design constraints.

GIVEN:

Design a rip rap lined channel to non-erosively convey the 5 year storm event.

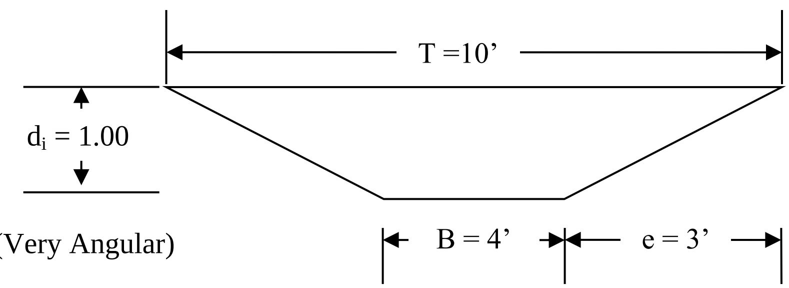
Q_5 = 17.4 cfs
3:1 side slopes (Z = 3)
S = 8% = 0.08
Trapezoidal Shape

REQUIRED:

Determine the required riprap D_{50} through an iterative process. Then compare the required D_{50} size to the trial D_{50} size. If D_{50} required < trial D_{50} then the rip rap size is adequate. However, a smaller more cost effective rip rap size should be considered if the trial $D_{50} \geq$ 110% of the required D_{50}.

SOLUTION:

Step 1: Q = 17.4 cfs
S = 8% = 0.08
$Z = \frac{e}{d} = 3$



Step 2: Trial D_{50} = 1.25’ (Very Angular)

Step 3:

d_i = 1.00 ft

$A = Bd + Zd^2$ = 7.00 sq ft
$T = B + 2dZ$ = 10.00 ft
$R = \frac{bd+Zd^2}{b+2d\sqrt{Z^2+1}}$ = 0.68 ft

d_a = A/T = 0.70 ft
$A_a = Bd + Zd^2$ = 4.27 sq ft
$T_a = B + 2dZ$ = 8.20 ft
$Ra = \frac{bd+Zd^2}{b+2d\sqrt{Z^2+1}}$ = 0.51 ft

Step 4: d_a/D_{50} = 0.56 ≤ 1.5 therefore use Equation 7.27-4

$$n = \frac{\alpha {d_a}^{1/6}}{\sqrt{g}\, f(Fr) f(REG) f(CG)} = 0.078$$

$$b = 1.14 \left(\frac{D_{50}}{T_a}\right)^{0.453} \left(\frac{d_a}{D_{50}}\right)^{0.814} = 0.303$$

$$v = Q/A_a = 4.075 \text{ ft/sec}$$

$$Fr = \frac{v}{\sqrt{g\, d_a}} = 0.858$$

$$f(Fr) = \left(\frac{0.28Fr}{b}\right)^{\log(0.755/b)} = 0.912$$

$$f(REG) = 3.434\left(\frac{T_a}{D_{50}}\right)^{0.492} b^{1.025\left(\frac{T_a}{D_{50}}\right)^{0.118}} = 7.363$$

$$f(CG) = \left(\frac{T_a}{d_a}\right)^{-b} = 0.474$$

Note: Subcritical flow, Froude Number (Fr) less than 1, which is desirable. Now check trial flow.

Step 5: $Q = \frac{1.49}{n} A_a R_a^{\frac{2}{3}} S^{\frac{1}{2}}$ = 14.72 cfs

This flow is not within 5% of 17.4 cfs; therefore return to step 3 and select a new depth (d_{i+1}).

Step 3(2): Using equation 7.27-2 obtain a new d_i (d_{i+1}):

$$d_{i+1} = d_i\left(\frac{Q}{Q_i}\right)^{0.4} = 1.07 \text{ ft}$$

d_{i+1} = 1.07 ft
$A = Bd + Zd^2$ = 7.72 sq ft
$T = B + 2dZ$ = 10.42 ft
$R = \frac{bd+Zd^2}{b+2d\sqrt{Z^2+1}}$ = 0.72 ft

$d_a = A/T$ = 0.74 ft
$A_a = Bd + Zd^2$ = 4.61 sq ft
$T_a = B + 2dZ$ = 8.44 ft
$Ra = \frac{bd+Zd^2}{b+2d\sqrt{Z^2+1}}$ = 0.53 ft

Step 4(2): d_a/D_{50} = 0.59 ≤ 1.5 therefore use Equation 7.27-4

$$n = \frac{\alpha d_a^{1/6}}{\sqrt{g}\, f(Fr) f(REG) f(CG)} = 0.080$$

$$b = 1.14\left(\frac{D_{50}}{T_a}\right)^{0.453}\left(\frac{d_a}{D_{50}}\right)^{0.814} = 0.313$$

$v = Q/A_a$ = 3.778 ft/sec

$$Fr = \frac{v}{\sqrt{g\, d_a}} = 0.774$$

$$f(Fr) = \left(\frac{0.28Fr}{b}\right)^{\log(0.755/b)} = 0.869$$

$$f(REG) = 3.434\left(\frac{T_a}{D_{50}}\right)^{0.492} b^{1.025\left(\frac{T_a}{D_{50}}\right)^{0.118}} = 7.746$$

$$f(CG) = \left(\frac{T_a}{d_a}\right)^{-b} = 0.466$$

Step 5(2): $Q = \frac{1.49}{n} A_a R_a^{\frac{2}{3}} S_f^{\frac{1}{2}}$ = 15.99 cfs

This flow is not within 5% of 17.4 cfs; therefore return to step 3 and select a new depth (d_{i+2})

Step 3(3): Using equation 7.27-2 obtain a new d_i (d_{i+2}):

$$d_{i+2} = d_{i+1}\left(\frac{Q}{Q_i}\right)^{0.4} = 1.11 \text{ ft}$$

d_{i+2} = 1.11 ft
$A = Bd + Zd^2$ = 8.14 sq ft
$T = B+2dZ$ = 10.66 ft
$R = \frac{bd+Zd^2}{b+2d\sqrt{Z^2+1}} = 0.74$ ft

d_a = A/T = 0.76 ft
$A_a = Bd + Zd^2$ = 4.80 sq ft
$T_a = B+2dZ$ = 8.58 ft
$Ra = \frac{bd+Zd^2}{b+2d\sqrt{Z^2+1}} = 0.54$ ft

Step 4(3): d_a/D_{50} = 0.61 ≤ 1.5 therefore use Equation 7.27-4

$$n = \frac{\alpha {d_a}^{1/6}}{\sqrt{g}\, f(Fr) f(REG) f(CG)} = 0.080$$

$$b = 1.14\left(\frac{D_{50}}{T_a}\right)^{0.453}\left(\frac{d_a}{D_{50}}\right)^{0.814} = 0.319$$

$v = Q/A_a$ = 3.624 ft/sec

$$Fr = \frac{v}{\sqrt{g\, d_a}} = 0.731$$

$$f(Fr) = \left(\frac{0.28Fr}{b}\right)^{\log(0.755/b)} = 0.847$$

$$f(REG) = 3.434\left(\frac{T_a}{D_{50}}\right)^{0.492} b^{1.025\left(\frac{T_a}{D_{50}}\right)^{0.118}} = 7.963$$

$$f(CG) = \left(\frac{T_a}{d_a}\right)^{-b} = 0.462$$

Step 5(3): $Q = \frac{1.49}{n} A_a {R_a}^{\frac{2}{3}} {S_f}^{\frac{1}{2}} = 16.75$ cfs

This flow is within 5% of 17.4 cfs, therefore go to Step 6.

Step 6: $R_e = \frac{\sqrt{gdS}\, D_{50}}{\nu} = 185117 = 1.85*10^5$

Note: The "d" used here is d_a + minimum freeboard of 0.5'

From Figure 7.27-4:

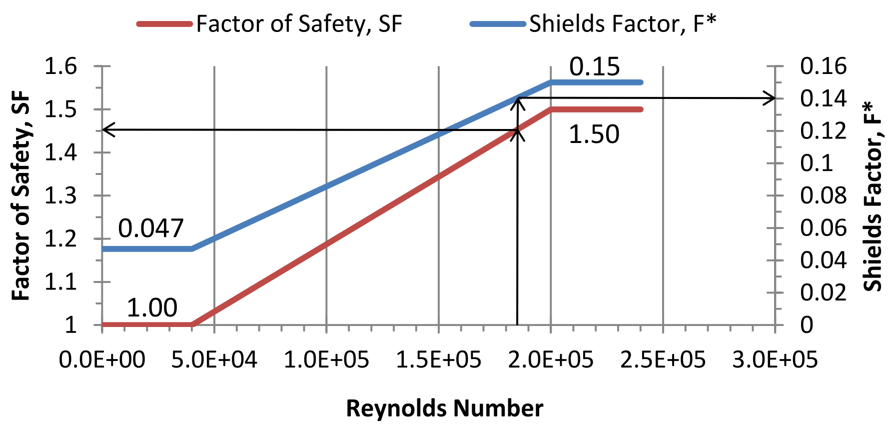
SF = 1.45

F* = 0.14

From Figure 7.27-4 interpolation or chart below:

SF = ((R_e – 40,000) * (3.125*10^{-6})) + 1
= ((1.85*10^5 – 40,000) * (3.125*10^{-6})) + 1= 1.45

F* = ((R_e -40000)*(6.4375*10^{-7})) + 0.047
= ((1.85*10^5 - 40000)*(6.4375*10^{-7}))+0.047 = 0.14



Reynolds Number, R_e	Shield's Parameter, F*	Factor of Safety, SF
$\leq 4x10^4$	0.047	1
$4x10^4 < R_e < 2x10^5$	Linear Interpolation	Linear Interpolation
$\geq 2x10^5$	0.15	1.5

Step 7: Since slope is between 5% and 10%, use both Equation 7.27-11 and Equation 7.27-12 and choose the larger outcome.

Equation 7.27-11:

$$D_{50} \geq \frac{SF\ d\ S}{F^*\left(\frac{\gamma_S}{\gamma}-1\right)} = 0.64 \text{ ft}$$

d = d_a + minimum freeboard of 0.5'
γ_s = specific weight of rock was assumed to be 165 lb/ft^3
γ = specific weight of water, 62.4 lb/ ft3

Equation 7.27-12:

$$D_{50} \geq \frac{SF\ d\ S\Delta}{F^*\left(\frac{\gamma_S}{\gamma}-1\right)} = 0.82 \text{ ft}$$

$$\tau_s = \gamma d_a S_o = 3.81 \text{ lb/ft}^2$$

$$\eta = \frac{\tau_s}{F^*\ (\gamma_s-\gamma)D_{50}} = 0.211$$

Note: The D_{50} used here is the trial D_{50} (1.25').

$$\beta = \tan^{-1}\left(\frac{\cos\alpha}{\frac{2\sin\theta}{\eta\tan\phi}+\sin\alpha}\right) = 16.33°$$

$\alpha = \tan^{-1}(S) = \tan^{-1}(0.08) = 4.57°$

$\theta = \tan^{-1}(1/Z) = \tan^{-1}(1/3) = 18.44°$

$\varphi = 42°$ (From Figure 7.27-5 using the trial D_{50} size (1.25') and Very Angular)

$$\Delta = \frac{K_1(1+\sin(\alpha+\beta))\tan\phi}{2\,(\cos\theta\tan\phi - SF\ \sin\theta\cos\beta)} = 1.284$$

$K_1 = 0.066Z + 0.67 = 0.066(3) + 0.67 = 0.868$

Note: $K_1 = .77\ (Z \leq 1.5)$
$= 0.066Z + 0.67\ (1.5 < Z < 5)$
$= 1.0\ (Z \geq 5)$

Therefore the required D_{50} size is 0.82 ft.

Step 8: Compare the required D_{50} to the trial size selected in Step 2. If the trial size is smaller than the required size, it is unacceptable for the design. Repeat the procedure from Step 2, selecting a larger trial size. If the trial size is larger than the required D_{50}, then the design is acceptable. However, if the required D_{50} is sufficiently smaller than the trial size, the procedure may be repeated from Step 2 with a smaller, more cost-effective stone size.

In the design example, the trial D_{50} is larger than the required D_{50} therefore the design is acceptable. However since it is significantly larger than the required D_{50}, return to Step 2 using the previous iteration's required D_{50} of 0.82 ft as the new trial D_{50}.

Step 2(2): Trial D_{50} = 0.82' (Very Angular)

Step 3(4):

$d_i = 1.00$ ft

$A = Bd + Zd^2 = 7.00$ sq ft

$T = B+2dZ = 10.00$ ft

$$R = \frac{bd+Zd^2}{b+2d\sqrt{Z^2+1}} = 0.68 \text{ ft}$$

$A_a = Bd + Zd^2 = 4.27$ sq ft

$T_a = B+2dZ = 8.20$ ft

$$Ra = \frac{bd+Zd^2}{b+2d\sqrt{Z^2+1}} = 0.51 \text{ ft}$$

$d_a = A/T = 0.70$ ft

Step 4(4): $d_a/D_{50} = 0.85 \leq 1.5$ therefore use Equation 7.27-4

$$n = \frac{\alpha {d_a}^{1/6}}{\sqrt{g}\ f(Fr) f(REG) f(CG)} = 0.065$$

$$b = 1.14\left(\frac{D_{50}}{T_a}\right)^{0.453}\left(\frac{d_a}{D_{50}}\right)^{0.814} = 0.353$$

$v = Q/A_a$ = 4.075 ft/sec

$f(REG) = 3.434 \left(\frac{T_a}{D_{50}}\right)^{0.492} b^{1.025\left(\frac{T_a}{D_{50}}\right)^{0.118}}$ = 10.29

$Fr = \frac{v}{\sqrt{g\, d_a}}$ = 0.858

$f(Fr) = \left(\frac{0.28Fr}{b}\right)^{\log(0.755/b)}$ = 0.881

$f(CG) = \left(\frac{T_a}{d_a}\right)^{-b}$ = 0.419

Step 5(4): $Q = \frac{1.49}{n} A_a R_a^{\frac{2}{3}} S_f^{\frac{1}{2}}$ = 17.56 cfs

This flow is within 5% of 17.4 cfs; therefore go to Step 6.

Step 6: $R_e = \frac{\sqrt{gdS}\, D_{50}}{v}$ = 118000 = $1.18*10^5$

Note: The "d" that is used here is d_a + minimum freeboard of 0.5'

From Figure 7.27-4:

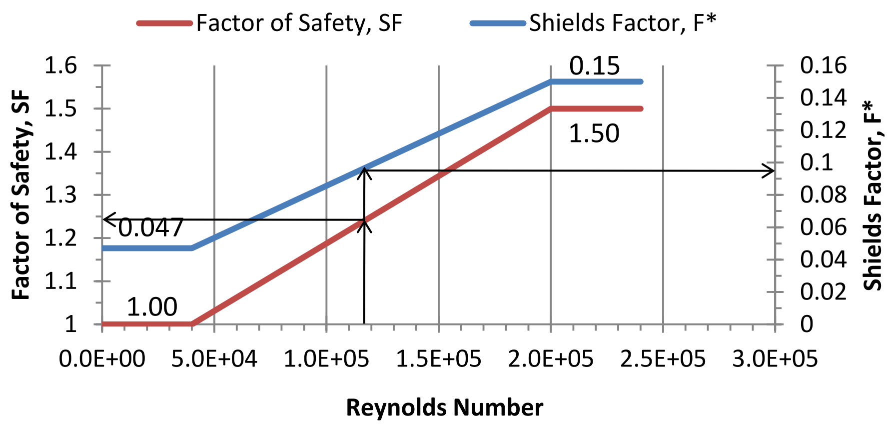
SF = 1.24

F* = 0.098

From Figure 7.27-4 interpolation:

SF = ((R_e – 40,000) * ($3.125*10^{-6}$)) + 1
= (($1.18*10^5$ – 40,000) * ($3.125*10^{-6}$)) + 1= 1.245

F* = ((R_e -40000)*($6.4375*10^{-7}$)) + 0.047
= (($1.18*10^5$ - 40000)*($6.4375*10^{-7}$)) + 0.047 = 0.098



Reynolds Number, R_e	Shield's Parameter, F*	Factor of Safety, SF
$\leq 4x10^4$	0.047	1
$4x10^4 < R_e < 2x10^5$	Linear Interpolation	Linear Interpolation
$\geq 2x10^5$	0.15	1.5

Step 7: Since slope is between 5% and 10% we must use both Equation 7.27-12 and Equation 7.27-13 and choose the larger outcome.

Equation 7.27-11:

$$D_{50} \geq \frac{SF\ d\ S}{F^* \left(\frac{\gamma_S}{\gamma} - 1\right)} = 0.75 \text{ ft}$$

d = d_a + minimum freeboard of 0.5'
γ_s = specific weight of rock was assumed to be 165 lb/ft^3
γ = specific weight of water, 62.4 lb/ ft3

Equation 7.27-12:

$$D_{50} \geq \frac{SF\ d\ S\Delta}{F^* \left(\frac{\gamma_S}{\gamma} - 1\right)} = 0.90 \text{ ft}$$

$$\tau_s = \gamma d_a S_o = 3.49 \text{ lb/ft}^2$$
$$\eta = \frac{\tau_s}{F^* (\gamma_s - \gamma) D_{50}} = 0.426$$

Note: The D_{50} that is used here is the trial D_{50} (0.82').

$$\beta = \tan^{-1}\left(\frac{\cos\alpha}{\frac{2\sin\theta}{\eta\tan\phi} + \sin\alpha}\right) = 29.55°$$

$\alpha = \tan^{-1}(S) = \tan^{-1}(0.08) = 4.57°$

$\theta = \tan^{-1}(1/Z) = \tan^{-1}(1/3) = 18.44°$

φ = 41.5° (From Figure 7.27-5 using the trial D_{50} size (1.25') and Very Angular)

$$\Delta = \frac{K_1(1+\sin(\alpha+\beta))\tan\phi}{2\,(\cos\theta\tan\phi - SF\ \sin\theta\cos\beta)} = 1.21$$

$K_1 = 0.066Z + 0.67 = 0.066(3) + 0.67 = 0.868$

Note: K_1 = .77 (Z ≤ 1.5)
= 0.066Z + 0.67 (1.5 < Z <5)
= 1.0 (Z ≥5)

Therefore the required D_{50} size is 0.90 ft.

Step 8: The trial D_{50} is smaller than the required D_{50} therefore the design is unacceptable. Return to Step 2 and use the previous iteration's required D_{50} of 0.90 ft as the new trial D_{50}.

Step 2(5): Trial D_{50} = 0.90' (Very Angular)

Step 3(5):

d_i = 1.00 ft	d_a = A/T = 0.70 ft
$A = Bd + Zd^2$ = 7.00 sq ft	$A_a = Bd + Zd^2$ = 4.27 sq ft
$T = B+2dZ$ = 10.00 ft	$T_a = B+2dZ$ = 8.20 ft
$R = \frac{bd+Zd^2}{b+2d\sqrt{Z^2+1}}$ = 0.68 ft	$Ra = \frac{bd+Zd^2}{b+2d\sqrt{Z^2+1}}$ = 0.51 ft

Step 4(5): d_a/D_{50} = 0.78 ≤ 1.5 therefore use Equation 7.27-4

$$n = \frac{\alpha d_a{}^{1/6}}{\sqrt{g}\ f(Fr)f(REG)f(CG)} = 0.068$$

$$b = 1.14\left(\frac{D_{50}}{T_a}\right)^{0.453}\left(\frac{d_a}{D_{50}}\right)^{0.814} = 0.341$$

$v = Q/A_a$ = 4.08 ft/sec

$$Fr = \frac{v}{\sqrt{g\ d_a}} = 0.858$$

$$f(Fr) = \left(\frac{0.28Fr}{b}\right)^{\log(0.755/b)} = 0.886$$

$$f(REG) = 3.434\left(\frac{T_a}{D_{50}}\right)^{0.492} b^{1.025\left(\frac{T_a}{D_{50}}\right)^{0.118}} = 9.540$$

$$f(CG) = \left(\frac{T_a}{d_a}\right)^{-b} = 0.432$$

Step 5(5): $Q = \frac{1.49}{n} A_a R_a{}^{\frac{2}{3}} S_f{}^{\frac{1}{2}}$ = 16.86 cfs

This flow is within 5% of 17.4 cfs, therefore go to Step 6.

Step 6(5): $R_e = \frac{\sqrt{gdS}\,D_{50}}{\nu}$ = 130000 = $1.30*10^5$

Note: The "d" that is used here is d_a + minimum freeboard of 0.5'

From Figure 7.27-4:

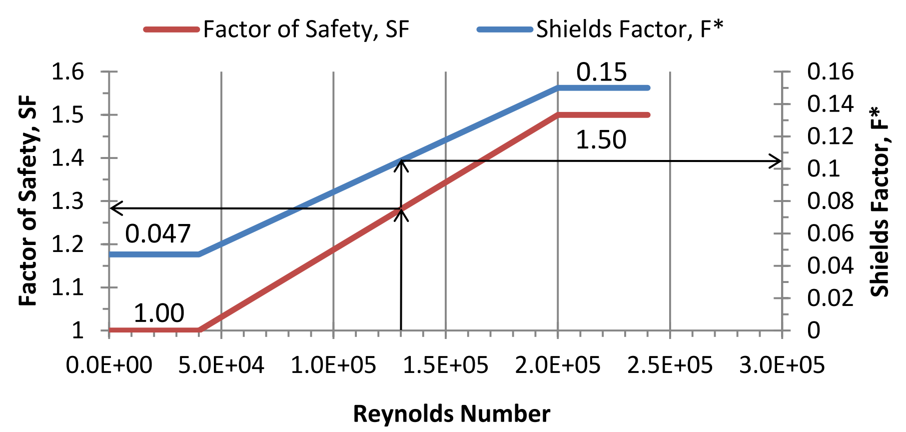
SF = 1.28

F* = 0.105

From Figure 7.27-4 interpolation:

SF = $((R_e - 40{,}000) * (3.125*10^{-6})) + 1$
$= ((1.30*10^5 - 40{,}000) * (3.125*10^{-6})) + 1 = 1.28$

F* = $((R_e - 40000)*(6.4375*10^{-7})) + 0.047$
$= ((1.30*10^5 - 40000)*(6.4375*10^{-7}))+0.047 = 0.105$



Reynolds Number, R_e	Shield's Parameter, F*	Factor of Safety, SF
$\leq 4x10^4$	0.047	1
$4x10^4 < R_e < 2x10^5$	Linear Interpolation	Linear Interpolation
$\geq 2x10^5$	0.15	1.5

Step 7(5): Since slope is between 5% and 10%,w use both Equation 7.27-11 and Equation 27-12 and choose the larger outcome.

Equation 7.27-11:

$$D_{50} \geq \frac{SF\ d\ S}{F^*\left(\frac{\gamma_S}{\gamma} - 1\right)} = 0.71 \text{ ft}$$

d = d_a + minimum freeboard of 0.5'
γ_s = specific weight of rock was assumed to be 165 lb/ft^3

γ = specific weight of water, 62.4 lb/ ft3

Equation 7.27-12:

$D_{50} \geq \frac{SF\ d\ S\Delta}{F^* \left(\frac{\gamma_S}{\gamma}-1\right)}$ = 0.86 ft

$\tau_s = \gamma d_a S_o$ = 3.494

$\eta = \frac{\tau_s}{F^*\ (\gamma_s-\gamma) D_{50}}$ = 0.361

Note: The D_{50} that is used here is the trial D_{50} (0.90').

$\beta = \tan^{-1}\left(\frac{\cos\alpha}{\frac{2\ \sin\theta}{\eta\tan\phi}+\sin\alpha}\right)$ = 26.03°

$\alpha = \tan^{-1}(S) = \tan^{-1}(0.08) = 4.57°$

$\theta = \tan^{-1}(1/Z) = \tan^{-1}(1/3) = 18.44°$

φ = 41.8° (From Figure 7.27-5 using the trial D_{50} size (1.25') and Very Angular)

$\Delta = \frac{K_1(1+\sin(\alpha+\beta))\tan\phi}{2\ (\cos\theta\tan\phi - SF\ \sin\theta\cos\beta)}$ = 1.21

K1 = 0.066Z + 0.67 = 0.066(3) + 0.67 = 0.868

Note: K_1 = .77 (Z ≤ 1.5)
= 0.066Z + 0.67 (1.5 < Z <5)
= 1.0 (Z ≥5)

Therefore the required D_{50} size is 0.86ft.

Step 8(5): **Specify Riprap with D_{50} = 12" = 1' for this channel.**

The trial D_{50} is slightly larger than the required D_{50} which is preferable. Ideally the trial D_{50} will be no more than 10% larger than the required D_{50}. One can then use this D_{50} size to specify the appropriate common riprap size, which in this case would be riprap with a D_{50} of 12" or 1'. The use of Excel is strongly recommended for performing these iterations.