Calculus | C

Calculus is designed for students interested in STEM-based careers and builds on the concepts studied in precalculus. The study of calculus on the high school level includes a study of limits, derivatives, and an introduction to integrals.

Calculus includes the following domains and clusters:

Limits of Functions
- Understand the concept of the limit of a function.

Behavior of Functions
- Describe the asymptotic and unbounded behavior of functions.

Continuity
- Develop an understanding of continuity as a property of functions.

Understand the Concept of the Derivative
- Demonstrate an understanding of the derivative.
- Understand the derivative at a point.

Computing and Applying Derivatives
- Apply differentiation techniques.
- Use first and second derivatives to analyze a function.
- Apply derivatives to solve problems.

Understanding Integrals
- Demonstrate understanding of a definite integral.
- Understand and apply the Fundamental Theorem of Calculus.

Calculate and Apply Integrals
- Apply techniques of antidifferentiation.
- Apply integrals to solve problems.

Mathematical Modeling

Mathematical Modeling is a Standard for Mathematical Practice (MP4) and a Conceptual Category. Specific modeling standards appear throughout the high school standards indicated with a star (★). Where an entire domain is marked with a star, each standard in that domain is a modeling standard.

Standards for Mathematical Practice

Being successful in mathematics requires the development of approaches, practices, and habits of mind that need to be in place as one strives to develop mathematical fluency, procedural skills, and conceptual understanding. The Standards for Mathematical Practice are meant to address these areas of expertise that teachers should seek to develop in their students. These approaches, practices, and habits of mind can be summarized as "processes and proficiencies" that successful mathematicians have as a part of their work in mathematics. Additional explanations are included in the main introduction of these standards.

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Literacy Standards for Mathematics

Communication in mathematics employs literacy skills in reading, vocabulary, speaking and listening, and writing. Mathematically proficient students communicate using precise terminology and multiple representations including graphs, tables, charts, and diagrams. By describing and contextualizing mathematics, students create arguments and support conclusions. They evaluate and critique the reasoning of others, analyze, and reflect on their own thought processes. Mathematically proficient students have the capacity to engage fully with mathematics in context by posing questions, choosing appropriate problem-solving approaches, and justifying solutions. Further explanations are included in the main introduction.

Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies. 2. Understand and use correct mathematical vocabulary. 3. Discuss and articulate mathematical ideas. 4. Write mathematical arguments.

Functions, Graphs, and Limits

Limits of Functions (F.LF)

Cluster Headings	Content Standards
A. Understand the concept of the limit of a function.	**C.F.LF.A.1** Calculate limits (including limits at infinity) using algebra. **C.F.LF.A.2** Estimate limits of functions (including one-sided limits) from graphs or tables of data. Apply the definition of a limit to a variety of functions, including piece-wise functions. **C.F.LF.A.3** Draw a sketch that illustrates the definition of the limit; develop multiple real-world scenarios that illustrate the definition of the limit.

Behavior of Functions (F.BF)

Cluster Headings	Content Standards
A. Describe the asymptotic and unbounded behavior of functions.	**C.F.BF.A.1** Describe asymptotic behavior (analytically and graphically) in terms of infinite limits and limits at infinity. **C.F.BF.A.2** Discuss the various types of end behavior of functions; identify prototypical functions for each type of end behavior.

Continuity (F.C)

Cluster Headings	Content Standards
A. Develop an understanding of continuity as a property of functions	**C.F.C.A.1** Define continuity at a point using limits; define continuous functions. **C.F.C.A.2** Determine whether a given function is continuous at a specific point. **C.F.C.A.3** Determine and define different types of discontinuity (point, jump, infinite) in terms of limits. **C.F.C.A.4** Apply the Intermediate Value Theorem and Extreme Value Theorem to continuous functions.

Derivatives

Understand the Concept of the Derivative (D.CD)

Cluster Headings	Content Standards
A. Demonstrate an understanding of the derivative.	**C.D.CD.A.1** Represent and interpret the derivative of a function graphically, numerically, and analytically. **C.D.CD.A.2** Interpret the derivative as an instantaneous rate of change. **C.D.CD.A.3** Define the derivative as the limit of the difference quotient; illustrate with the sketch of a graph. **C.D.CD.A.4** Demonstrate the relationship between differentiability and continuity.
B. Understand the derivative at a point.	**C.D.CD.B.5** Interpret the derivative as the slope of a curve (which could be a line) at a point, including points at which there are vertical tangents and points at which there are no tangents (i.e., where a function is not locally linear). **C.D.CD.B.6** Approximate both the instantaneous rate of change and the average rate of change given a graph or table of values. **C.D.CD.B.7** Write the equation of the line tangent to a curve at a given point. **C.D.CD.B.8** Apply the Mean Value Theorem. **C.D.CD.B.9** Understand Rolle's Theorem as a special case of the Mean Value Theorem.

Computing and Applying Derivatives (D.AD)

Cluster Headings	Content Standards
A. Apply differentiation techniques.	**C.D.AD.A.1** Describe in detail how the basic derivative rules are used to differentiate a function; discuss the difference between using the limit definition of the derivative and using the derivative rules. **C.D.AD.A.2** Calculate the derivative of basic functions (power, exponential, logarithmic, and trigonometric). **C.D.AD.A.3** Calculate the derivatives of sums, products, and quotients of basic functions. **C.D.AD.A.4** Apply the chain rule to find the derivative of a composite function. **C.D.AD.A.5** Implicitly differentiate an equation in two or more variables **C.D.AD.A.6** Use implicit differentiation to find the derivative of the inverse of a function.

B. Use first and second derivatives to analyze a function.	**C.D.AD.B.7** Relate the increasing and decreasing behavior of f to the sign of f' both analytically and graphically. **C.D.AD.B.8** Use the first derivative to find extrema (local/relative and global/absolute). **C.D.AD.B.9** Analytically locate the intervals on which a function is increasing, decreasing, or neither. **C.D.AD.B.10** Relate the concavity of f to the sign of f" both analytically and graphically. **C.D.AD.B.11** Use the second derivative to find points of inflection as points where concavity changes. **C.D.AD.B.12** Analytically locate intervals on which a function is concave up, concave down, or neither. **C.D.AD.B.13** Relate corresponding characteristics of the graphs of f, f', and f". **C.D.AD.B.14** Translate verbal descriptions into equations involving derivatives and vice versa.
C. Apply derivatives to solve problems.	**C.D.AD.C.15** Model rates of change, including related rates problems. In each case, include a discussion of units. **C.D.AD.C.16** Solve optimization problems to find a desired maximum or minimum value. **C.D.AD.C.17** Use differentiation to solve problems involving velocity, speed, and acceleration. **C.D.AD.C.18** Use tangent lines to approximate function values and changes in function values when inputs change (linearization).

Integrals

Understanding Integrals (I.UI)

Cluster Headings	Content Standards
A. Demonstrate understanding of a definite integral.	**C.I.UI.A.1** Define the definite integral as the limit of Riemann sums and as the net accumulation of change. **C.I.UI.A.2** Write a Riemann sum that represents the definition of a definite integral. **C.I.UI.A.3** Use Riemann sums (left, right, and midpoint evaluation points) and trapezoid sums to approximate definite integrals of functions represented graphically, numerically, and by tables of values.
B. Understand and apply the Fundamental Theorem of Calculus.	**C.I.UI.B.4** Recognize differentiation and antidifferentiation as inverse operations. **C.I.UI.B.5** Evaluate definite integrals using the Fundamental Theorem of Calculus. **C.I.UI.B.6** Use the Fundamental Theorem of Calculus to represent a particular antiderivative of a function and to understand when the antiderivative so represented is continuous and differentiable. **C.I.UI.B.7** Apply basic properties of definite integrals (e.g. additive, constant multiple, translations).

Calculate and Apply Integrals (I.AI)

Cluster Headings	Content Standards
A. Apply techniques of antidifferentiation.	**C.I.AI.A.1** Find antiderivatives that follow directly from derivatives of basic functions (power, exponential, logarithmic, and trigonometric). **C.I.AI.A.2** Use substitution of variables to calculate antiderivatives (including changing limits for definite integrals). **C.I.AI.A.3** Find specific antiderivatives using initial conditions.
B. Apply integrals to solve problems.	**C.I.AI.B.4** Use a definite integral to find the area of a region. **C.I.AI.B.5** Use a definite integral to find the volume of a solid formed by rotating a region around a given axis. **C.I.AI.B.6** Use integrals to solve a variety of problems (e.g., distance traveled by a particle along a line, exponential growth/decay).